Effect of Layer Thickness on Natural Convection in a Square Enclosure Superposed by Nano-Porous and Non-Newtonian Fluid Layers Divided by a Wavy Permeable Wall

Mohammed Y. Jabbar\textsuperscript{1}  Saba Y. Ahmed\textsuperscript{1,*}  Hameed K. Hamzah\textsuperscript{1}  Farooq Hassan Ali\textsuperscript{1}  Salwan Obaid Waheed Khafaji\textsuperscript{1}

\textsuperscript{1}Mechanical Engineering Department, College of Engineering, University of Babylon, Babylon, Iraq.

mohyousif 2269@gmail.com
farooq_hassan77@yahoo.com
hameedkadhem1977@gmail.com

*Corresponding author: saba_ya@yahoo.com
sw9p4@mail.missouri.edu
Mobile:009647801275760

Abstract--- The aim of this work is to investigate numerically the influence of layer thickness and wavy interface wall on the heat transfer and fluid flow inside a differentially heated square enclosure occupied with two portions, Ag/water nanofluid-porous medium and non-Newtonian substance, respectively. The numerical computations have been carried out for Rayleigh number Ra= 10^{3} to 10^{5}, number of undulation N= 1 to 4 (four cases) Darcy number Da= 10^{-1} to 10^{-5}, volume fraction ϕ= 0 to 0.2 non-Newtonian index n= 0.6 to 1.4 and the Prandtl number of water Pr= 6.2, for different interface location S=0.25, 0.5 and 0.75. The governing equations were solved numerically by using finite element techniques based Glarkine approach solver to take out an expression of streamlines and isotherms. The output results were compared with previous work and it was observed that a good formal equivalent between the results. Results demonstrated that as power law index "n" increases the average Nusselt number decreases due to high viscosity and shear force of pseudoplastic fluid. Consequently, layer thickness has a major impact while the number of undulation has a minor impact on the heat transfer rate across the layers.

Index Term--- Natural convection; Sinusoidal vertical interface; Nanofluid porous medium; Non-Newtonian fluid; Prandtl number; Average Nusselt number.

<table>
<thead>
<tr>
<th>Nomenclature</th>
<th>N</th>
<th>Number of undulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cp</td>
<td>Specific heat at constant pressure (KJ/kg.K)</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>Rate of strain-tensor</td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>Gravitational acceleration (m/s\textsuperscript{2})</td>
<td></td>
</tr>
<tr>
<td>K</td>
<td>Thermal conductivity (W/m.K)</td>
<td></td>
</tr>
<tr>
<td>L</td>
<td>Dimensionless Length and Height of the cavity</td>
<td></td>
</tr>
<tr>
<td>P</td>
<td>Dimensionless pressure</td>
<td></td>
</tr>
<tr>
<td>Pr</td>
<td>Prandtl number (ν/α)</td>
<td></td>
</tr>
<tr>
<td>Ra</td>
<td>Rayleigh number (gβfL\textsuperscript{3} ΔT/νfαf)</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>Temperature (K)</td>
<td></td>
</tr>
<tr>
<td>Da</td>
<td>Darcy numbers</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>Layer thickness</td>
<td></td>
</tr>
<tr>
<td>Θ</td>
<td>Dimensionless temperature (T-Tc/ΔT)</td>
<td></td>
</tr>
<tr>
<td>φ</td>
<td>Nanoparticle volume fraction (%)</td>
<td></td>
</tr>
<tr>
<td>Ψ</td>
<td>Dimensional stream function (m\textsuperscript{2}/s)</td>
<td></td>
</tr>
<tr>
<td>ε</td>
<td>Permeability of porous medium</td>
<td></td>
</tr>
<tr>
<td>Ψ</td>
<td>Dimensionless stream function</td>
<td></td>
</tr>
<tr>
<td>β</td>
<td>Volumetric coefficient of thermal expansion (K\textsuperscript{-1})</td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>Dynamic viscosity (kg.s/m)</td>
<td></td>
</tr>
<tr>
<td>ρ</td>
<td>Density (kg/m\textsuperscript{3})</td>
<td></td>
</tr>
</tbody>
</table>
1. INTRODUCTION

The wavy interface, undulation number and the layer thickness were important roles that govern the energy transformation and fluid motion in a very wide practical application. The main applications related with wavy interface included the thermal insulation treatments, a depuration of sludgy water by separating the sinter which passing horizontally over the wavy plate and the dispersion of cooling of emigrated heat. The undulation number investment in a very important field to increase the efficiency of heat transfer, such as radiator design in cooling and heating in electronic equipment. Thereafter the thickness of the layer has attracted the attention of many engineering applications such as solidification and melting process and cooling tower filling design. After spasmodically meditation, the cavity wall in the literature either straight or curved lines. The straight wall cavities defined as square, rectangular or trapezoidal and the curved wall cavities founded similitude to wavy sinusoidal or semicircular. Besides that, the cylindrical and irregular solid shapes cavities were arranged at the end of the literature.

The rectangular and square shaped cavities were arranged from [1 to 10]. Al-Zamily [1 and 2] the square and partitioned cavities were performed numerically. The first [1], the partition lines are vertical forming three adjacent layers filled with a nanofluid and one porous layer in the middle. The results indicate that the average Nusselt number increases with increases of the thickness of the middle porous layer. The second [2], the layers are two and a horizontal one of them is porous and the other is nanofluid with the presence of a magnetic field. It was founded at high Rayleigh number, and Darcy number increases the effect of Hartmann number on the Nusselt number increase. Similar to the three vertical layers that were mentioned in [1], Gao et al. [3] used a modified Lattice Boltzmann method to study a conjugate heat transfer in these three mediums again. Chamkha and Ismael [4] consolidated two vertical porous layers in a rectangular cavity occupied by a nanofluid, to study numerically the natural conduction. They concluded that at a certain Rayleigh number and Nusselt number is the maximum the critical porous layer thickness was denoted. Iman [5] utilized a heatline visualization method to explain the heat transport path for buoyancy-driven flow inside a rectangular porous cavity filled with nanofluids. The results show that the Cu-water nanofluid having the higher rates of heat transfer in all cases. Such as cavity of reference [5], Homman [6] add the vector of energy flux determination as the viscosity variations with temperature was considered. It was found that at a certain temperature of evaluation of fluid properties evaluation, was a function of the shape parameter of porous media and other considered parameters. The same rectangular porous cavities of references [5 and 6] but with different boundary conditions were investigated by Yasin et al. [7]. The sinusoidal profile of temperatures on the bottom cavity wall was considered and the others were adiabatic. It was noticed that the amplitude increasing of the sinusoidal temperature distribution the rate of heat transfer increases. Muthamilselvan and Sureshkumar [8], proposed a square porous cavity with lid-driven filled with nanofluid and different heat sources. The results reveal a reduction in the flow intensity as the solid volume fraction increases. Sheikholeslamy et al. [9] Presented a square porous cavity with centered hot circular obstacle filled with nanofluid and subjected to Lorentz forces to study the forced convection by using the Lattic Boltzmann technique. The results show that the Nusselt number increases of Darcy and Rayleigh numbers. A triangular abstraction was fixed at the lower left corner of a nanofluid porous cavity by Muneer et al. [10] To investigate numerically the heat transfer and entropy generation. They predicted the entropy generation increases with nanoparticles addition.

As a complement of the straight wall cavities the trapezoidal cavities were presenting from [11 to 16]. Alsabary et al. [11 and 12] studied numerically the heatline visualization of the natural convection of natural convection in trapezoidal cavities filled partially with nanofluid porous layer and partially with non-Newtonian fluid layer [12]. The results showed that the variation of cavity inclination angle effect on the heat transfer rate. Yasin [13] performed a numerical study to analyze the behavior of heat.
transportation and the fluid flow due to natural convection in a trapezoidal cavity with the presence of a horizontal solid partition and filled with fluid-saturated porous medium. It was observed that the conduction mode became dominant inside the cavity for higher partition thickness. Ratish and Bipin [14] coupled a numerical finite element method with a Krylov subspace solver to study the natural convection in a trapezoidal porous cavity. Nusselt number was noted to increase with increasing of Rayleigh and Grashof numbers. Tanmay et al. [15] performed a numerical investigation of natural convection in trapezoidal, porous cavities with uniform and non-uniform heated bottom wall was illustrated using heatlines. They found that minimum local natural convection was near the corners of the bottom wall. Mohammad et al. [16] investigated numerically the natural convection inside a trapezoidal cavity occupied by a nanofluid. The results show at low value of Rayleigh number $10^4$ the average Nusselt number decreases.

The curved walls cavities founded similitude to a sinusoidal wave from [17 to 26]. Sheikholeslami and Sadoughi [17] demonstrated the effect of the magnetic field on a cavity filled with a nanofluid in porous media. Results proved that the values of Nusselt number decreases when the Lorentz forces increase. Sheikholeslami et al. [18 and 19] used the same cavity but with differ boundary conditions, then the numerical study were occurring under the effect under the magnetic field and electrical field, respectively. The results approved that the heat transfer rate reduces with rise of Hartmann number and the convection mode was boosted by electric field, respectively. Sheikholeslami and Shamlooei [20] simulated the effect of magnetic field on the flow of nanofluid in the cavity with permeable medium. The influence of nanoparticles shape was taken into account. The results detected the velocity of the fluid and Nusselt number was decreasing due to increase in Hartmann number. Sheikholeslami and Shehzad [21] studied the nanofluid migration using Darcy model inside a permeable medium by control volume based finite element method. The results illustrated that the distribution of the isotherms becomes more complex with increasing of buoyancy force. Same cavity, but with different boundary conditions subjected to a magnetic field was investigated by Sheikholeslami and Houman [22]. The results show that the heat transfer rate reduces with increasing of buoyancy force. Minh et al. [23] investigated the effect of a horizontal wavy interface on the natural convection inside a nanofluid porous medium by using ISPH formulation. It was depicted that the average Nusselt number reduces due to increase in amplitude, height and the number of undulation of the sinusoidal interface. The natural convection in a permanent cavity was investigated numerically with two adiabatic horizontal wavy walls and the other sides subjected a variable temperature by Sherment and Pop [24]. It was found that the average Nusselt number were increasing with Lewis number decreases. Khalil et al. [25] investigated numerically the natural convection inside a porous cavity with a left, vertical, isothermal and wavy side wall. This investigation shows that the wavy surface amplitude and undulation number affect isotherms. Salam [26] by using heatline visualization technique, investigated numerically the natural convection and the generation of entropy inside porous, two sides wavy walls tilted cavity with the presence of a magnetic field. Moreover, the results depicted that the generation of entropy due to the horizontal magnetic field were higher than the vertical.

The curved walls similitudes to a semi-circular cavity were presented from [27 to 31]. Ali et al. [27] has been analyzed numerically the conjugate unsteady, natural convection and entropy generation inside a semi-circular permanent cavity. It was concluded that the happening of entropy generation along the internal interface of solid-porous at high Rayleigh number. Sheikholeslami and Ganji [28] examine the impact magnetic field external source on Fe3O4 nanofluid in a porous medium. It was indicated that the increase in Hartmann number leads to decrease in transformation of nanofluid. Sheikholeslami and Shehzad [29] proposed a simulation of convective flow in the porous nanofluid gap between hot elliptical inner wall and cold circular outer wall. The outputs results demonstrated that the maximum streamline values enhance with an increase of Rayleigh number. The semi-circular cavity filled with a nanofluid and faced with heating source, was carried out by Ali and Amin [30] to investigate the natural convection and entropy generation. it was clear from the results the rate of heat transfer enhanced with rise of Rayleigh number. Same cavity that subjected to a heat flux in reference [30] was subjected to a magnetic field by Ali [31] to check its main effect on natural convection. The results detected the rate of heat transfer diminution with a progress of Hartmann number. An annulus porous cavity with conjugate heat transfer was studied by Irfan et al. [32]. Same as vertical annulus porous cavity, but with different boundary conditions have been analyzed numerically by Sankar et al. [33]. It was denoted that the lower half of the inner wall was the perfect place for heater to produce highest heat transfer rate. T-shaped inclined cavity with various types of
nanofluids subjected to a uniform heat flux source and the magnetic field was examined numerically by Ahmed et al. [34]. It was concluded the mean Nusselt number decreases with increases of the heat source length and Hartmann number.

Finally, Ahmed et al. [35] studied natural convective heat transfer within trapezoidal cavity having a horizontal porous media partition and filled with TiO$_2$-water nanofluid. Their results showed that the maximum value of the percentage enhancement of heat transfer was 4.22% occurred at porous position equal to 0.5. The wavy word came in the literature either refers to the external cavity walls or the interface between two or more adjacent mediums. Therefore, it was obvious from the above literature the rarity of information’s concerning with the effect of vertical sinusoidal interface and its location on the fluid migration and heat transportation inside a cavity with partially permeable medium and partially occupied by a non-Newtonian fluid as well as, the complexity due to geometry and numerical simulation, therefore, this work was execution.

2. Physical model explanation

The square cavity is represented by four cases shown schematically in Fig. 1. The cavity with the height of (L) is heated from the left vertical wall and is cooled from the right wall whereas the other two walls are considered adiabatic. This cavity is filled with two layers of nanofluid porous medium and non-Newtonian fluid. These layers are vertically separated by the sinusoidal vertical interface where the interface shape is changed depending on undulation numbers, namely one undulations (case1), two undulations (case2), three undulations (case3) and four undulations (case4). Furthermore, some assumptions are offered:

1. The fluid flow is incompressible, steady and laminar.
2. The density in the buoyancy term depends only on temperature, which is approximated by the standard Boussinesq model, whereas other physical properties of non-Newtonian nanofluid are taken constant.
3. The thermal conductivity of the nanofluid is assumed to equal the effective thermal conductivity of the porous medium.
4. Thermal equilibrium is taken between the water and silver nanoparticles.
5. The rigid boundaries of cavity are impermeable (no momentum exchange) and the velocity components are zero because of no-slip condition, while the boundary of sinusoidal interface is assumed to be permeable.
6. The equations of extended Darcy–Brinkman model and the energy transport are adopted to depict the flow of non-Newtonian fluid and the heat transfer process in the Ag-water nanofluid and porous layer.

3. Governing equations and boundary conditions

The two dimensions governing equations of the porous medium-nanofluid layer are:

\[
\begin{align*}
\text{Continuity} & \quad \frac{\partial u_{pn}}{\partial x} + \frac{\partial v_{pn}}{\partial y} = 0 \\
\text{X – momentum} & \quad u_{pn} \frac{\partial u_{pn}}{\partial x} + v_{pn} \frac{\partial u_{pn}}{\partial y} = -\frac{1}{\rho_{pn}} \frac{\partial p}{\partial x} + \mu_{pn} \left( \frac{\partial^2 u_{pn}}{\partial x^2} + \frac{\partial^2 u_{pn}}{\partial y^2} \right) \\
\text{Y – momentum} & \quad \frac{\partial \phi_{pn}}{\partial t} + u_{pn} \frac{\partial \phi_{pn}}{\partial x} + v_{pn} \frac{\partial \phi_{pn}}{\partial y} = 0
\end{align*}
\]
\[
u_{pn} \frac{\partial v_{pn}}{\partial x} + v_{pn} \frac{\partial u_{pn}}{\partial y} = -\frac{1}{\rho_{pn}} \frac{\partial p}{\partial y} + \mu_{pn} \left( \frac{\partial^2 v_{pn}}{\partial x^2} + \frac{\partial^2 v_{pn}}{\partial y^2} \right) + \left( \rho \beta \right)_{pn} g(T_{pn} - T_c) - \frac{\mu_{pn} v_{pn}}{\rho_{pn} \varepsilon} \tag{3}
\]

Energy
\[
u_{pn} \frac{\partial T_{pn}}{\partial x} + v_{pn} \frac{\partial T_{pn}}{\partial y} = \alpha_{pn} \left( \frac{\partial^2 T_{pn}}{\partial x^2} + \frac{\partial^2 T_{pn}}{\partial y^2} \right) \tag{4}
\]

The two-dimensional governing equations for the non-Newtonian fluid are:

Continuity
\[
\frac{\partial u_{nN}}{\partial x} + v_{nN} \frac{\partial u_{nN}}{\partial y} = 0 \tag{5}
\]

X-momentum
\[
u_{nN} \frac{\partial u_{nN}}{\partial x} + v_{nN} \frac{\partial u_{nN}}{\partial y} = -\frac{1}{\rho_{nN}} \frac{\partial p}{\partial y} + \frac{1}{\rho_{nN}} \left( \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} \right) + \rho_{nN} \beta_{nN} g(T_{pn} - T_c) \tag{6}
\]

Y-momentum
\[
u_{nN} \frac{\partial v_{nN}}{\partial x} + v_{nN} \frac{\partial v_{nN}}{\partial y} = -\frac{1}{\rho_{nN}} \frac{\partial p}{\partial y} + \frac{1}{\rho_{nN}} \left( \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} \right) + \rho_{nN} \beta_{nN} g(T_{pn} - T_c) \tag{7}
\]

Energy
\[
u_{nN} \frac{\partial T_{nN}}{\partial x} + v_{nN} \frac{\partial T_{nN}}{\partial y} = \alpha_{nN} \left( \frac{\partial^2 T_{nN}}{\partial x^2} + \frac{\partial^2 T_{nN}}{\partial y^2} \right) \tag{8}
\]

Non-Newtonian fluid is used which follow the power law model, therefore the shear stress tensor is Ostwald-DeWaele [36]
\[
\tau_{ij} = 2\mu_a D_{ij} = \mu_a \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \tag{9}
\]

Where \( D_{ij} \) is the rate of strain tensor for the two-dimensional Cartesian coordinate and \( \mu_a \) is the dimensional Cartesian coordinate as:
\[
\mu_a = \sigma \left[ \left( \frac{\partial u_{nN}}{\partial x} \right)^2 + \left( \frac{\partial u_{nN}}{\partial y} \right)^2 \right] + \left( \frac{\partial v_{nN}}{\partial x} + \frac{\partial v_{nN}}{\partial y} \right)^2 \right] \frac{n-1}{n} \tag{10}
\]

Where \( \sigma \) and \( n \) are power-law model constants. \( (\sigma) \) is the consistency coefficient and \( (n) \) is the power-law index. Where \((n<1)\), it represents a pseudoplastic such as condensed milk, toothpaste, ketchup, etc. and \((n>1)\), it represents a dilatant fluid such as cornstarch and water mixtures. When \( (n=1) \) a Newtonian fluid is obtained. The two layers (porous-nano fluid layer) is considered separately by sinusoidal permeable interface with corresponding joint conditions at that interface. These conditions include continuity, normal stress, shear stress, heat rate, and temperature.

These conditions can be expressed as equations [11-15] at the permeable interface.
\[
T_{pn} \bigg|_{x=l^+} = T_{nN} \bigg|_{x=l^+} \tag{11}
\]

\[
k_{pn} \frac{\partial T_{pn}}{\partial x} \bigg|_{x=l^+} = k_{nN} \frac{\partial T_{nN}}{\partial x} \bigg|_{x=l^+} \tag{12}
\]

\[
u_{pn} \bigg|_{x=l^{-}} = u_{nN} \bigg|_{x=l^+}, v_{pn} \bigg|_{x=l^{-}} = v_{nN} \bigg|_{x=l^+} \tag{13}
\]

\[
p_{pn} \bigg|_{x=l^{-}} = p_{nN} \bigg|_{x=l^+} \tag{14}
\]

\[
\mu_{pn} \left( \frac{\partial v_{pn}}{\partial x} + \frac{\partial u_{pn}}{\partial y} \right) \bigg|_{x=l^{-}} = \mu_{nN} \left( \frac{\partial v_{nN}}{\partial x} + \frac{\partial u_{nN}}{\partial y} \right) \bigg|_{x=l^+} \tag{15}
\]

Equation (15) denotes addition of the shear stress corresponding condition [37]. To convert the systems of equations (1-10) to the dimensionless form, one can suppose the following assumptions:
\[
X = \frac{x}{L}, Y = \frac{y}{L}, U = \frac{L u}{a_{fl}}, V = \frac{L v}{a_{fl}}, P = \frac{L^2 p}{\rho a_{fl}^2}, \theta_{pn} = \frac{T-T_c}{T_{pn}-T_c}, \theta_{nN} = \frac{T-T_c}{T_{nN}-T_c} \tag{16}
\]

The set of governing dimensionless equations for the porous-nano fluid layer become:

Continuity
\[
\frac{\partial U_{pn}}{\partial X} + \frac{\partial V_{pn}}{\partial Y} = 0 \tag{17}
\]

X-momentum
The non-dimensional equation of the heat function for the porous-nanofluid layer can be expressed as [41]:

\[
\frac{\partial^2 H}{\partial X^2} = U_\theta - \alpha_{pn} \frac{\partial \theta}{\partial X} - \frac{\partial H}{\partial Y} = V\theta - \alpha_{pn} \frac{\partial \theta}{\partial Y} \tag{30}
\]
Which produces the following equation.
\[
\frac{\partial^2 H}{\partial X^2} + \frac{\partial^2 H}{\partial Y^2} = \frac{\partial}{\partial Y}(V\theta) - \frac{\partial}{\partial X}(U\theta)
\] (31)

The non-dimensional equation of the heat function for the non-Newtonian fluid layer expressed as[42]:
\[
\frac{\partial H}{\partial Y} = U\theta - \frac{\partial H}{\partial X}, \frac{\partial H}{\partial X} = V\theta - \frac{\partial H}{\partial Y}
\] (32)

Which produce the following equation.
\[
\frac{\partial^2 H}{\partial X^2} + \frac{\partial^2 H}{\partial Y^2} = \frac{\partial(U\theta)}{\partial Y} - \frac{\partial(V\theta)}{\partial X}
\] (33)

The Neuman boundary condition for heat function for both thermal vertical walls from the equation (32) and the normal deviation (\(\hat{n}\nabla H\)) are defined as below.
\[
\hat{n}\nabla H = 0 \text{ at the thermal vertical walls}
\] (34)
\[
\hat{n}\nabla H = \frac{k_{pn}}{k_{fl}} \int_0^1 \frac{\partial \theta_{pn}}{\partial X} |_{x=0} dY \text{ at the adiabatic top wall}
\] (35)

The adiabatic bottom wall may be expressed by the Dirichelet boundary condition attained from the equation (32), which reduced to the \(\frac{\partial H}{\partial X} = 0\) for and adiabatic wall. A mention amount of heat function is supposed as at \(X=0, Y=0\) and therefore heat function \((H=0)\) is accurate for \(Y=0\). \(H=\alpha_{pn}/\alpha\) \(Nu_{pn}\) is found from equation (32) at \(X=0, Y=0\). The total amount of heat transfer rate within the enclosure can expressed by the average Nusselt number equations along the heated wall.
\[
Nu_{ave} = \int_0^1 \left[ \left( k_{pn}/k_{fl} \right) \frac{\partial \theta_{pn}}{\partial X} \right] dY
\] (36)

4. Numerical procedure and code validation

Dimensionless governing equations with dimensionless boundary conditions in the earlier section have been given are described using finite element method. The following assumption were considered, laminar flow, two-dimensional, steady state, incompressible and Newtonian fluid with Boussing approximation is used to describe the buoyancy effect. Galerkin finite element least-square method is used to discretization the partial differential equation and to ensure the stability. The nonlinear equations which produce from Galarkine finite element method solved by using the damped Newtonian method. The following assumptions are used incompressible fluid, two dimensional, laminar flow, steady state and the Poisson’s equation are used for the stream function
\[
\frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2} = \frac{\partial U}{\partial Y} - \frac{\partial V}{\partial X}
\] (37)

This study examined at various Darcy numbers, volume fraction, Prandtl number and power-law index. The current program was verified for grid independence by computing the average Nusselt number along the left hot wall as reported in table 2. It was found that a grid size \((180*180)\) confirms a grid independence explanation. To ensure the accurate results, the current program results are compared with a published article by [11].

In this work, rectangular elements with mapped mesh shown in Fig. 2, is descrtrized in the trapezoidal cavity. Rectangular finite elements of variable order are taken for every term of the partial differential equation. The error for the relative convergence solution for each of the variables in governing equation convenience the following criteria
\[
\left| \frac{E^{i+1} - E^i}{E^i} \right| \leq 10^{-6}, \text{ where } i \text{ signify the iteration number. The results are validated in Fig. 3, as the stream and isothermal lines have a good convenient with the published article by [11].}
\]

The weak formula of the dimensionless governing equations is achieved, after approaching flow variables are replaced in the dimensionless governing equations. Residual Approach Consequences (RAC) and the weighted average of residual approach will be compelled to equal to zero as in Equation below. For more information on the finite element approach can be gotten in [43, 44].
\[
\int_{\text{Domain}} W \ast (\text{RAC}) dA = 0
\] (38)
5. Results and Discussion
The numerical solution was carried out to investigate the effect of wavy vertical interface on the natural convection in a differentially heated cavity filled with two mediums, porous on the left and non-Newtonian fluid on the right. For all calculations, the pure water with Pr=6.2 was a base fluid, Ra=10^3 to 10^5, Da=10^{-1} to 10^{-3} and \( \phi \)=0 to 0.2

1- Effect of Rayleigh number

Figs. 4 to 1 show the effect of the Rayleigh number on the streamlines and isotherms. It was clear from the figures of the streamlines the circulation was obviously enhanced with Ra put up to 10^5. Consequently, the centrifugal force increases pushing the convergent streamlines from the core of the large one vortex to the boundaries in the right non-Newtonian medium. Additionally, the weakness of circulation strength at low Ra, defeat the fluid to permeate through the left porous medium, so the streamlines were very difficult to penetrate. While at high Ra, the streamlines penetrated significantly at the left medium. Commonly, the streamlines values were rises with Ra rises. The results of streamlines contours showed, after mixing the nanoparticles, red lines (\( \phi \)=0.05), the stream lines magnitude (\( \Psi_{\text{max, nf}} \) and \( \Psi_{\text{min, nf}} \)) was overall rises if compared with a pure water, green lines (\( \phi \)=0).

The isotherms contours display different heat flow schemes in the porous and non-Newtonian mediums as Ra increase. At low Ra=10^4, the isotherms trend to be arranged vertically not overall cavity, but especially at left porous medium, therefore, the isotherms demonstrated to be transitional case and unclaimed to say a pure conduction heat transfer regime. While, at the right non-Newtonian medium, the isotherms patterns obliquity is perfect for the dominated convection heat transfer flow. As the Ra increases to 10^5, the vortex circulation intensity was increasing. Therefore, the obliquity of the isotherms lines gotten clear more, thereby, the convection mode was the dominant on the left and right mediums of the cavity. Due to low permeability at DA=10^5 the conduction regime keeps dominating in the left porous medium.

Fig. 12 shows the relation between Darcy number and average Nusselt number on the left hot wall. It was obvious the average Nusselt number was increasing with increasing of Ra from 10^3 to 10^5 overall values of Da. Uniquely case it was observed at S=0.75, the average Nusselt number doesn’t enhance with Ra increases at low Da= 10^{-5} to 10^{-4}, but rises from Da=10^{-3} to 10^{-1} due to dominant conduction heat transfer at low permeability in the left porous medium.

2- Effect of Darcy number:

Figs. 4 to 11 shows the streamlines and isotherms for Ra=10^4 and 10^5, Da=10^{-3} and 10^{-5} and \( \phi \)=0 and 0.05. The streamlines contours explain the decreasing Da from 10^{-3} to 10^{-5} prevent the nanofluid penetration through the porous medium due to the permeability decreases. In most cases this weakens the vortices circulation. The thermal behavior changes the mechanism of heat transfer from weak convection to conduction with decreasing Da in the left porous medium. But the isotherms distribution on the right non-Newtonian medium becomes more than before and maintain to convection mode.

At low Da=10^{-5}, the high hydrodynamic resistances displayed by the left porous medium, this lead to low nanofluid penetration in this medium. The non-Newtonian fluid enclosed by left porous medium and cold right side wall enjoys with dominated convection heat transfer mode.

Generally, at high Da=10^{-3} the effect of nanoparticles addition \( \phi \)=0.05 (red lines) on the streamlines contours, figs. 4 and 6, was obvious if it was compared with low Da=10^{-5}, figs. 8 and 10, because the nanoparticles adding increase the fluid revolving in the two mediums occupied the cavity. Inversely, at low Da=10^{-5} the streamlines patterns in the left porous medium were almost appear as a result of permeability reduction and the nanoparticles addition \( \phi \)=0.05 affect slightly on the non-Newtonian fluid part. The augmentation of water thermal conductivity due to nanoparticle additive enhances the conductivity of it, the augment with heat transfer rate.

Fig. 12 explains the relation between Da and average Nu for different cases. As Da built up, the permeability was increased in the left porous medium and the domination heat transfer was convection, hence more heat transfer rate. Generally, a directly proportional relation between these two parameters was denoted.

3- Effect of the interface location:

Figs. 4 to 11 explain the width S effect of the porous layer on the stream function and isotherms for Ra=10^4 and 10^5, Da=10^{-3} and 10^{-5} and \( \phi \)=0 to 0.05. The fluid revolving intensity decreases dramatically with the left porous medium width increases from 0.25 to 0.75. Whenever, the interface aboard from the right wall the porous layer expanded, therefore, the stream function values were decreases and the streamlines compressed toward the cold side wall. The addition of nanoparticles enhanced the stream function in both porous and non-Newtonian
Generally, when the width of the left porous layer increases, the isothermlines inclination reduces (change from horizontal distribution to vertical). Consequently, the conduction regime was overcome and the convection reduces gradually with width of porous medium increases from 0.25 to 0.75.

Fig. 12 shows the relation between average Nu and Da. After focusing, the descent of the average Nu values at any Ra, were detected from S=0.25 to 0.75. The cause of this descent was reducing of fluid circulation that is reflected on the nature of heat transfer.

4- Effect of the interface undulation number (F):

The undulation number of the wavy interface is interesting geometrical criterion were demonstrated. Figs. 8 and 10 show the cavity stream function and isotherms for Ra=10^5, 10^6, Da=10^{-5} and S=0.75. The contour maps of streamline show the core of a single elongated vortex at F=1 located nearest to the upper adiabatic wall. As F increases from 2 to 4, the streamlines obeys and follow the sinusoidal shape of the interface and splits into 2, 3 and 4 individual, small and weak vortices that was contribute to reduce the stream function values. It should be noticed, at F=3 and 4 the length of the interface becomes longer (more contact area) allow to some of streamlines to penetrate slightly through the left porous layer. But at S=0.25 and 0.5, there was no dramatic influence occurred on streamlines only it was follow the interface shape too, and the lonely dominates vortex became smaller at S=0.5.

For other cases figs. 4 and 6 when Ra=10^4 and 10^5 and Da=10^{-3} there was no important change that was draw attention were observed. In respect to the isotherms, due to increase with number of undulation from 1 to 4, the isotherms patterns bending were reduces and a large interface area that was increasing with F, attempt to obrudles the conduction regime through the non-Newtonian fluid medium.

If we consider the Da=10^4, S=0.25 to 0.75 and Ra=10^3 to 10^5 of fig. 12, two things were noticed, the first one was the average Nu were decreases with increasing of F from F=1 (case 1) to F=4 (case 4) when S=0.25 and 0.5, because of the idleness of non-Newtonian fluid circulation. The second, at S=0.75, the maximum average Nu obtained when F=3 (yellow lines), the others were compacted to each other and almost don’t change with varying of F.

Fig. 13 show the relation between average Nu and volume fraction \( \phi \) at a constant Ra=10^5, Da=10^{-3}, n=0.7, F=1 to 4 and S=0.25 to 0.75. The average Nu increases at S=0.25 with volume fraction increases because of the fluid thermal conductivity increase. Inversely, when the S becomes 0.5 and 0.75 the behavior of the relation was changed from increases to decreases due to the circulation idleness, and then the convection regime reduces gradually as the sinusoidal interface nears the right vertical cold wall. As well as, it was clear from fig. 13, when S=0.25, the value of average Nu were reduces as F increases except F=3 (case 3) the average Nu values were higher than the other cases, due to the matching between the concavity of the interface and the streamlines contours. Hence, more uniform fluid motion and uniform heat transfer rate than the other cases (1, 2 and 4).

As the porous layer thickness increase the uniform fluid circulation and heat transfer regime occurs too in the (cases 2 and 3) and (cases 2, 3 and 4) when S=0.5 and S=0.75 fig. 13, respectively. Therefore, when S=0.75 the values of the average Nu for (cases 2, 3 and 4) were convergent forming almost one line parallel up to the average Nu of (case 1), due to circulation velocity was slow down and governs the situation as the thickness of the porous medium increases. Under these conditions the increases of S don’t effect on the values of average Nu.

Two important things can be concluded from fig. 14 show the relation between the average Nu and power law index n. The first, the average Nu was decreases with rising the n because as n increases the circulation intensity slows down, due to high viscosity and shear force of pseudo-plastic flow. Therefore, decreases heat transfer rate and average Nu. The second, clearly, the convection heat transfer enhanced at high Da (0.0001 to 0.1) which lead to increase the average Nu. But at low Da (0.00001 to 0.0001) the convection remains weak, accordingly, the average Nu decrease slightly.

Finally, it was appeared from fig. 14, the increasing of F from 1 to 4 have not effect on average Nu values, then the four figures of the four cases were similar.

Conclusion

1- The streamlines values increased with Ra increase.
2- The strength of streamlines enhanced with addition of nanoparticles.

3- At lowest $Da=10^{-3}$ the porous medium becomes like a solid material, hence, plug the permeability ahead the nanofluid to penetrate through the porous medium, consequently, the conduction heat transfer was dominating.

4- The nanoparticle additive enhance the stream function at high $Da=10^{-3}$ and overall augment the heat transfer flow through the cavity.

5- Generally, whenever the width of the porous layer increase the conduction mode overcome and the convection mode recedes.

6- With increasing the number of undulations F, the conduction regime to force through the non-Newtonian fluid circulation nears the interface.

7- As power law index n increases, the average Nu decreases due to high viscosity and shear force of pseudo plastic fluid.

REFERENCES


Table I

Thermo-physical of Ag/water nanofluid

<table>
<thead>
<tr>
<th>Properties</th>
<th>Water</th>
<th>Ag</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_p$ (J/kg.K)</td>
<td>4179</td>
<td>235</td>
</tr>
<tr>
<td>$\rho$ (kg/m$^3$)</td>
<td>997.1</td>
<td>10500</td>
</tr>
<tr>
<td>$k$ (w/m.K)</td>
<td>0.6</td>
<td>429</td>
</tr>
<tr>
<td>$\beta \times 10^3$ (W/m.K)</td>
<td>21</td>
<td>5.4</td>
</tr>
</tbody>
</table>
Table II  
Grid testing for Average Nusselt number at different mesh number for Pr = 6.2, Ra = 10^{5}; Da = 10^{-3}; n = 0.7; φ=0.05, Case 4.

<table>
<thead>
<tr>
<th>Test number</th>
<th>Domain elements</th>
<th>Boundary elements</th>
<th>Average Nusselt number</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 × 50</td>
<td>2000</td>
<td>230</td>
<td>3.9278</td>
</tr>
<tr>
<td>70 × 70</td>
<td>4900</td>
<td>350</td>
<td>3.9603</td>
</tr>
<tr>
<td>90 × 90</td>
<td>8100</td>
<td>450</td>
<td>3.9662</td>
</tr>
<tr>
<td>100 × 100</td>
<td>10000</td>
<td>500</td>
<td>3.9680</td>
</tr>
<tr>
<td>120 × 120</td>
<td>14400</td>
<td>600</td>
<td>3.9703</td>
</tr>
<tr>
<td>140 × 140</td>
<td>19600</td>
<td>700</td>
<td>3.9718</td>
</tr>
<tr>
<td>160 × 160</td>
<td>25600</td>
<td>800</td>
<td>3.9728</td>
</tr>
<tr>
<td>180 × 180</td>
<td>32400</td>
<td>900</td>
<td>3.9730</td>
</tr>
</tbody>
</table>

Fig. 2. Mesh Distribution of the Model
Fig. 3. Comparision of the present study and A.I. Alsabery et al [5] for Streamlines and Isotherms contours, for $Ra = 10^5, Da = 10^{-3}$; $n = 0.7$; $S = 0.5$, $\varphi = 16.7, Pr=13.4$ and $4.4$; $\varphi=0$ (green line) and $\varphi=0.05$ (red lines).
\( \phi_{\max} = 3.5339, \quad \phi_{\min} = -6.0655, \quad \phi_{\max} = 0 \)

\( \phi_{\max} = 3.7752, \quad \phi_{\min} = -4.4567, \quad \phi_{\max} = 0 \)

\( \phi_{\max} = 1.4127, \quad \phi_{\min} = -2.0045, \quad \phi_{\max} = 0 \)

\( \phi_{\max} = 1.3099, \quad \phi_{\min} = -1.9942, \quad \phi_{\max} = 0 \)

\( \phi_{\max} = 3.7031, \quad \phi_{\min} = -5.8605, \quad \phi_{\max} = 0 \)

\( \phi_{\max} = 3.8457, \quad \phi_{\min} = -4.2896, \quad \phi_{\max} = 0 \)

\( \phi_{\max} = 1.3921, \quad \phi_{\min} = -2.0045, \quad \phi_{\max} = 0 \)

\( \phi_{\max} = 1.3006, \quad \phi_{\min} = -4.4567, \quad \phi_{\max} = 0 \)

\( \phi_{\max} = 1.2286, \quad \phi_{\min} = -3.5339, \quad \phi_{\max} = 0 \)

\( \phi_{\max} = 1.3428, \quad \phi_{\min} = -5.8823, \quad \phi_{\max} = 0 \)

\( \phi_{\max} = 1.3333, \quad \phi_{\min} = -3.4333, \quad \phi_{\max} = 0 \)

\( \phi_{\max} = 1.3534, \quad \phi_{\min} = -3.5534, \quad \phi_{\max} = 0 \)

**Fig. 4.** Streamlines contours: \( \phi = 0 \) (green line) and \( \phi = 0.05 \) (red lines).
Fig. 5. Isotherms contours: $T$, for $Ra = 10^4; Da = 10^{-3}; n = 0.7, Pr=6.2$ for different positions of porous surface and four cases; $\phi= 0$ (green line) and $\phi= 0.05$ (red lines).
\[ \psi_{\min} = -10.47, \psi_{\max} = 0 \]
\[ \psi_{\min} = -10.59, \psi_{\max} = 0.0032 \]

Fig. 6. Streamlines contours: \( \phi = 0 \) (green line) and \( \phi = 0.05 \) (red lines).

N=1

\( S=0.25 \)

\( S=0.5 \)

\( S=0.75 \)
Figure (7). Isotherms contours: $Ra = 10^5; Da = 10^{-3}; n = 0.7; Pr=6.2$ for different positions of porous surface and four cases; $\phi= 0$ (green line) and $\phi= 0.05$ (red lines).
Fig. 8. Streamlines contours: \( \psi = 0 \) (green line) and \( \psi = 0.05 \) (red lines).
Fig. 9. Isotherms contours for $Ra = 10^4$, $Da = 10^{-5}$; $n = 0.7$, $Pr=6.2$ for different positions of porous surface and four cases; $\phi = 0$ (green line) and $\phi = 0.05$ (red lines).
Fig. 10. Streamlines contours: , for \( \text{Ra} = 10^5; \text{Da} = 10^{-5}; n = 0.7, \text{Pr}=6.2 \) for different positions of porous surface and four cases; \( \phi = 0 \) (green line) and \( \phi = 0.05 \) (red lines).
Fig. 11. Isotherms contours: for $Ra = 10^5$, $Da = 10^{-5}$, $n = 0.7$, $Pr=6.2$ for different positions of porous surface and four cases; $\phi = 0$ (green line) and $\phi = 0.05$ (red lines).
Fig. 12. Average Nusselt Number along hot wall with Darcy number with different Rayleigh number and four cases.

Fig. 13. Average Nusselt number along hot wall with Volume Fraction with Four Cases.
Fig. 14. Average Nusselt Number along hot wall with Power-law index with different Darcy number and four cases.