Design of Anti-Windup Compensation Scheme for Steer-by-Wire System Subject to Time Delays and Actuator Saturation

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Abstract—An effective anti-windup scheme plays an important role to ensure the reliability and robustness of time critical steer-by-wire (SbW) system. This paper addresses a new SbW model formulation with the integration of anti-windup scheme approach which is designed to ensure stability of SbW system in the presence of time delays and actuator saturation. A steering system model is developed as the first step to pave the way for the development of the anti-windup scheme. The measurement of road wheel angle is used as a feedback in the designing process of the anti-windup compensator. Next, coupled with controller used in the ShW systems, an anti-windup compensator designs begin with the formation of augmented state matrix, where the stability condition is expressed in the form of linear matrix inequalities (LMIs). The adaptation of the vector valued of dead zone nonlinearity in the LMI design is introduced to reduce the unwanted saturation effect at the control input. The compensator parameters are obtained by solving a set of LMI conditions which is developed based on Lyapunov-Krasovskii candidate function. The efficacy of the proposed method is obtained by solving a set of LMI conditions which is developed based on Lyapunov-Krasovskii candidate function. The simulation run under various time delays conditions with saturated actuator demonstrated that the proposed method results in good system performance and fulfill control system requirement.

Index Term—Anti-windup, Linear matrix inequalities, Steer-by-wire, Time delays

I. INTRODUCTION

In recent years, the automobile industries have been working on adapting Drive-by-Wire (DbW) technology in the production of modern vehicles. The DbW system is integration of electronic sensors, controllers and actuators that is to replace hydraulic and mechanical subsystems such as steering, suspension and braking systems. This technology offers numerous advantages; mainly enhanced passengers’ safety significantly and improved driving convenience and overall performance of the vehicle by reducing power consumption [1-4]. Steer-by-Wire (SbW) system is a class of DbW systems where electronic actuators are replacing the conventional mechanical interface between the steering wheel at the driver side and the front wheels of the vehicle. The obvious advantages it offers are the reduction of the overall vehicle weight, decrease of noise and vibration, removal of environmental hazardous hydraulic fluids and energy saving. In addition to that, the SbW systems can also increase the adaptation of the steering feel and improve steering maneuverability. This feature has made the SbW system suitable for active steering control and practical to be implemented for autonomous self-driving vehicle [5-7].

The active steering capability, which is the ability to change the driver’s steering input to improve maneuverability and stability, has received considerable attentions from the academic researchers and automotive industrial experts. Important work by [8] suggests active steering use to improve direction stability and vehicle handling by integrating variable torque distribution control. To achieve improved yaw rate tracking in low to mid-range lateral acceleration, the researchers designed active front and rear steering via sliding mode controller (SMC). In [9], they proposed an observer-based analytical redundancy for SbW systems to reduce the total number of redundant road wheel angle sensors while maintaining high level of reliability. Then, a fault detection and isolation algorithm was developed by using a majority voting scheme to sustaining safe drivability. In [10], a non-linear adaptive dynamic surface sliding control is proposed to improve vehicle handling and path tracking simultaneously via SbW systems. Through this method, the adaptive robust controller not only improves both vehicle handling and path tracking, but also reduces the chattering problem when an uncertainty in the tires’ cornering stiffness occurs.

As the technology of SbW have adapted the Networked Control System (NCS) configuration, complexity in term of control configuration increases due to time induced delays resulted from data exchange over network. As reported in [11], the delay of NCS-SbW system is analyzed and PID controller is implemented in discrete form to compensate the time delays and maintaining control performance in the cases of different network speed and vehicle velocity. Furthermore,
stabilization of SbW systems with bounded delays in the control input and system states is proposed in [12]. The stability condition is expressed in linear matrix inequalities (LMIs) and it shows that the SbW system maintains its stability under some bounded time delays. In [13], the communication impact on quality of service in SbW system is evaluated where Controller Area Network (CAN) bus is used as a network. Under different communication factors such as transmission delay and network faults, a methodology is proposed to evaluate the vehicle steering behavior. Work in [14] proposed a robust fuzzy tracking controller based on Takagi-Sugeno (T-S) to ensure a resilient steering performance under network time delays and the nonlinear controller is solved in terms of LMI.

A part from the delay, actuator saturation of SbW is also a crucial issue to be overcome, as reported by [15-17]. In [15], the properties of saturation in the steering actuator were studied where from the simulation and experiment results, the steering angle rate actuator saturation form a major limitation of the system performance. Then, a new steering controller for vehicles operational with 4-wheel SbW that can track reference sideslip and yaw rate signals is proposed in [16]. The work also includes an anti-windup scheme, which allows the controller to perform adequately in the presence of actuators saturation. In [17], an LMI approach to robust vehicle steering controller is presented that consist of multi-objective optimization problem of system stabilization, actuator saturation and time delay.

Due to the physical limitations of steering actuator, not every required motor voltage input can ideally be generated. Therefore, through this proposed method the main advantage is to allow the control law to generate bounded control input that prevent the steering actuator saturation. Additionally, from the literatures, in spite of their importance, it is acknowledged that a little accomplishment has been made to account time delays and actuator saturation problem in SbW system in one research, which contribute to another advantage of this method.

This paper outlines the development of an anti-windup compensation methodology for the SbW system under condition time delays and actuator saturation. This paper is organized as follows. Section 2 describes the plant model dynamic of the SbW system and preliminary result. In section 3, the anti-windup controller for SbW system is designed based on Lyapunov-Krasovskii stability theory. Section 4 presents a selection of simulations results, along with discussion of the controller and system response behavior under various conditions. Section 5 concludes this paper.

II. PROBLEM FORMULATION

A. Model of SbW System

A SbW system model mostly incorporates a steering wheel, a set of steering wheel angle sensors, an electronic control units (ECUs), a road wheel actuator (DC motors) connected to the rack and pinion assembly and a set of road wheel angle sensors. In this work the control objective is to control the dynamics of the steering rack where the road wheel angle is tracking the desired steering wheel angle. The steering wheel angle sensors measure the instantaneous values and transmitted these values to ECUs via communication network. An electronic actuator that is a part of the system receives input signal from ECUs and steers the front wheel by following the drivers will. Figure 1 shows comparison of conventional steering system and SbW system where it can be seen that the steering wheel of the SbW system is not mechanically attached to the road wheels.

![Fig. 1. Comparison of conventional steering system and SbW system.](image1)

The SbW needs to meet the closed-loop feedback control requirement that is to preserve system stability and the illustration to describe the SbW control system configuration with anti-windup scheme is shown in Figure 2, where $\theta_s$ is the desired steering angle and $\theta_r$ is the road wheel angle.

![Fig. 2. The general SbW control system with anti-windup scheme.](image2)

In this work, the steering rack model of SbW system consists of a DC motor that rotates the pinion and rack to the
desired angle. Electrical current supplied to the dc motor generates required torque to overcome friction and self-aligning torque to produce angle movement of road wheel resulting in a change of vehicle dynamics. Hence, the rotation of the road wheel satisfies the following dynamic equation as in [9, 11]:

\[ J_r \ddot{\theta}_r + b_r \dot{\theta}_r + \tau_a + \tau_f = \eta K_{tr} i_r \]  

(1)

where \( J_r \) is moment of inertia of the road wheel, \( b_r \) is viscous damping coefficient, \( \theta_r \) is road wheel angle, \( \tau_a \) is self-aligning torque, \( \tau_f \) is friction torque, \( \eta \) is steering ratio, \( K_{tr} \) is motor torque constant and \( i_r \) is motor current. The road wheel angle \( \theta_r \) is the output and the control variable of the system.

The motor current can be described by:

\[ i_r \dot{L}_r + K_{er} \dot{\theta}_r + R_r i_r = V_r \]  

(2)

where \( L_r \) is motor inductance, \( K_{er} \) is electromagnetic force constant, and \( R_r \) is motor resistance. The input of the system is the voltage source \( V_r \) that is applied to the DC motor.

From (1) the steering rack dynamic equation can be re-written as:

\[ \dot{\theta}_r = -\frac{b_r}{J_r} \dot{\theta}_r + \frac{\eta K_{tr}}{J_r} i_r - \frac{1}{J_r} \tau_a - \frac{1}{J_r} \tau_f \]  

(3)

and from (2) the motor current equation can be re-written as:

\[ \dot{i}_r = -\frac{K_{er}}{L_r} \dot{\theta}_r - \frac{R_r}{L_r} i_r + \frac{1}{L_r} V_r \]  

(4)

The self-aligning torque \( \tau_a \) is due to the tire contact forces acting on the steering system to resist steering away from the straight-ahead position. While the friction force \( \tau_f \) is due to Coulomb friction force. Both torque parameters are given by the following equations.

\[ \tau_a = -C^a \alpha_F(t_p + t_m) \]  

(5)

\[ \tau_f = g t_p \mu W_f \text{sgn}(\dot{\theta}_r) \]  

(6)

where \( C^a \) is front tire cornering coefficient, \( \alpha_F \) is front tire slip angle, \( t_p \) is tire pneumatic trail, \( t_m \) is tire mechanical trail, \( g \) is gravity acceleration, \( \mu \) is friction coefficient and \( W_f \) is front tire weight.

The sideslip angle of tires and the vehicle motion and stability during turning maneuver can be observed from vehicle dynamic model. Based on single-track vehicle model, the linearized vehicle dynamic model of vehicle-body sideslip angle \( \beta \) and yaw rate \( r \) can be presented as the following equations [11].

\[ \dot{\beta} = -\frac{C^g \beta - C^a \dot{\theta}_r}{m v} \beta + \left( -1 + \frac{b C_R^g}{m v^2} \right) r \]  

\[ + \frac{C^g}{m v} \frac{\dot{\theta}_r}{\theta_r} \]  

(7)

\[ \dot{r} = \frac{b C_R^g - a C^g}{l_x} \beta + \left( -\frac{a^2 C_R^g}{l_x v} - b^2 C^g \right) r \]  

\[ + \frac{a C^g}{l_x} \frac{\dot{\theta}_r}{\theta_r} \]  

(8)

where \( C^g \) is rear tire cornering coefficient, \( m \) is vehicle mass, \( v \) is vehicle longitudinal velocity, \( a \) is distance from the front tire to the vehicle’s center of gravity (CoG), \( b \) is distance from the rear tire to the vehicle’s CoG and \( l_x \) is vehicle moment of inertia. Note that, equations (7) and (8) is valid based on the following considerations [11].

- Friction force in the x-direction are negligible when the vehicle in not braking.
- The left and right steering angles are same, such that \( \theta_L = \theta_R = \theta_r \).
- Vehicle is symmetry.
- Front tire contact force in the longitudinal and lateral direction for both left and right tires are same.

Considering that the sideslip angles is small such that approximately less than 4°, the slip angles of front-tire \( \alpha_F \) and rear-tire \( \alpha_R \) can be approximate as the following [11].

\[ \alpha_F = \beta + \frac{a}{v} r - \theta_r \]  

(9)

\[ \alpha_R = \beta - \frac{b}{v} r \]  

(10)

By substituting (9) into (5); the self-aligning torque equation now become:

\[ \tau_a = -C^a \alpha_F(t_p + t_m) \beta - \frac{a C^g}{v} (t_p + t_m) \theta_r \]  

(11)

Then, substitute (11) into (3); the steering rack dynamic equation can be re-written as:

\[ \dot{\theta}_r = \frac{C^g (t_p + t_m)}{J_r} \beta + \frac{a C^g}{J_r v} (t_p + t_m) r \]  

\[ - \frac{C^g (t_p + t_m)}{J_r} \theta_r + \frac{b_r}{J_r} \dot{\theta}_r + \frac{\eta K_{tr}}{J_r} i_r - \frac{1}{J_r} \tau_f \]  

(12)

Thus, the steering rack model and linearized vehicle dynamics model can be represented in a state-space form as:
\[ \dot{x}(t) = A_p x(t) + B_p u(t) \]
\[ y(t) = C_p x(t) \]  
\[ \text{(13)} \]

where

\[ x = [\beta \; r \; \theta_r \; i_r]^T, y = [V_r \; \tau_r]^T \]

\[ A_p = \begin{bmatrix} A_1 & A_2 \\ \begin{bmatrix} -C_p g - C_p g \frac{mv}{bC_p g - aC_p g} \\ bC_p g - aC_p g \frac{mv^2}{bC_p g - aC_p g} \\ I_x \\ 0 \\ C_p (t_p + t_m) \end{bmatrix} & \begin{bmatrix} -1 + \frac{bC_p g - aC_p g}{mv^2} \\ I_x v \\ 0 \\ aC_p g (t_p + t_m) \end{bmatrix} \\ \begin{bmatrix} J_r \\ 0 \\ C_a g \\ \frac{mv}{aC_a g} \\ I_x \\ 0 \\ \frac{C_p g (t_p + t_m)}{J_r} \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} & \begin{bmatrix} 1 \\ 0 \end{bmatrix} \end{bmatrix} \]

\[ B_p = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{J_r} \end{bmatrix} \quad C_p = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \]  
\[ \text{(14)} \]

This model representation is in line with the work in [9, 11, 18] and the illustration to describe the state variables and parameters of SbW system related to the physical system are shown in Figure 3.

\[ \text{Fig. 3. State variables and parameters of SbW vehicle system.} \]

\[ \text{B. Problem Statement and Preliminary Result} \]

Consider the SbW system subjected to time delay in control input that can be represented as follows:

\[ \dot{x}(t) = A_p x(t) + B_p u(t - \tau) + B_\omega \omega(t) \]
\[ y(t) = C_p x(t) \]
\[ z(t) = D_p x(t) + D_\zeta u(t) \]  
\[ \text{(15)} \]

where \( x(t) \in \mathbb{R}^n \) is the state variables, \( u(t) \in \mathbb{R}^m \) is the control input, \( \omega(t) \in \mathbb{R}^p \) is the disturbance, \( y(t) \in \mathbb{R}^q \) is the measured output and \( z(t) \in \mathbb{R}^p \) is the regulated output. \( A_p, B_p, B_\omega, C_p, D_p \) are known constant real matrices.

The delay \( \tau \) is assumed to be an unknown but bounded. Also, the control input is supposed to be constrained as follows:

\[ -u_0(t) \leq u(t) \leq u_0(t) \]  
\[ \text{(16)} \]

The disturbance vector \( \omega(t) \) is defined as \( \omega(t) \in \mathcal{L}_2 \). For some scalar \( \delta \), \( 0 \leq \frac{1}{\delta} \leq \infty \), therefore, the disturbance \( \omega(t) \) can be bounded to be:

\[ \| \omega(t) \|^2 = \int_0^\infty \omega^T(t)\omega(t)dt \leq \frac{1}{\delta} \]  
\[ \text{(17)} \]

By considering the SbW system in (15), it can be written that the stabilizing controller in the form of:

\[ \dot{x}_c(t) = A_c x_c(t) + B_c y(t) \]
\[ y_c(t) = C_c x_c(t) + D_c y(t) \]  
\[ \text{(18)} \]

where \( x_c(t) \) is the controller state and \( y_c(t) \) is the controller output. The controller input is defined as \( u_c = y(t) \). Also, \( A_c, B_c, C_c \) and \( D_c \) are known constant real matrices with appropriate dimensions.

Taking into account the saturation signal, when actuator saturation is present in SbW system; the control signal of the system can be represented as:

\[ u(t - \tau) = \text{sat}(y_c(t - \tau)) \]  
\[ \text{(19)} \]

where \( \text{sat}(y_c(t - \tau)) = \text{sign}(y_c(t - \tau)) \min\{|y_c(t - \tau)|, u_0\} \)

The vector valued of dead zone nonlinearity denoted as \( \psi(y_c(t)) \) is the difference between the applied control signal and the controller output signal, such that:

\[ \psi(y_c(t)) = y_c(t) - \text{sat}(y_c(t - \tau)) \]  
\[ \text{(20)} \]

Based on (20), then, there exist two conditions, which are;

1. No occurrences of saturation:
\[ \psi(y_c(t)) = y_c(t) - \text{sat}(y_c(t - \tau)) = 0, \quad \text{(C. 1)} \]
\[ u = y_c \]

2. With the present of saturation:
\[ \psi(y_c(t)) = y_c(t) - \text{sat}(y_c(t - \tau)) \neq 0, \quad (C.2) \]
\[ u = \text{sat}(y_c) \]

\[ \psi(y_c(t)) \] serves as the input to the controller. When the saturation of voltage occurs in the feedback system of SbW, the system behaves as an open-loop system. Then, the control input becomes unbounded and the input signal injected to the system increases excessively beyond nominal operating range. To moderate the unwanted consequences of windup, the anti-windup compensator is proposed, therefore, an additional signal is added to the control signal from the anti-windup compensator to bound the control input. Consequently, the newly augmented controller can be re-arranged such that:
\[ \dot{x}_c(t) = A_c x_c(t) + B_c y(t) - E_c \psi(y_c(t)) \]
\[ y_c(t) = C_c x_c(t) + D_c y(t) \]

(21)

Let the interconnection between the SbW system and the controller to be: \[ \xi(t) = [x(t)^T \ x_c^T(t)]. \] Therefore, the following augmented matrices are derived from (15) and (21).

\[ \dot{\xi}(t) = \mathcal{A} \xi(t) - (\mathcal{B} + \mathcal{K} \mathcal{C}_w(t)) \psi(\mathcal{K} \xi(t)) + \mathcal{B}_\omega \omega(t) \]
\[ z(t) = \mathcal{C}_z \xi(t) - \mathcal{D}_z \psi(\mathcal{K} \xi(t)) \]

(22)

and the augmented state and matrices are defined as:

\[ \xi(t) = \begin{bmatrix} x(t) \\ x_c(t) \end{bmatrix}, \quad \mathcal{A} = \begin{bmatrix} A_p + B_p D_c C_p & B_p C_c \\ B_c C_p & A_c \end{bmatrix}, \]
\[ \mathcal{B} = \begin{bmatrix} B_p \\ 0 \end{bmatrix}, \quad \mathcal{K} = \begin{bmatrix} D_c C_p & C_c \end{bmatrix}, \]
\[ \mathcal{B}_\omega = \begin{bmatrix} B_\omega \end{bmatrix}, \quad \mathcal{C}_z = \begin{bmatrix} C_z + D_z D_c C_p & D_z C_c \end{bmatrix}, \]
\[ \mathcal{D}_z = D_z \]

(23)

In order to arrive at the main result of this work, the following lemmas will be used.

**Lemma 1** [19]. For any proper dimensioned matrices \( R_i > 0, N_{ij}, N_{ji} \) where \( i = 1, 2 \) and a scalar \( \tau \), the following inequality holds:
\[ - \int_{t-\tau}^{t} \dot{\xi}_i^T(s) R_i \dot{\xi}(s) ds \leq 2 \zeta_i(t) Y_i \dot{\xi}_i(t) \]
\[ + \tau \dot{\xi}_i^T(t) Z_i \dot{\xi}_i(t) \]

(24)

where

\[ Y_i = \begin{bmatrix} N_{ii} & -N_{i1} \\ N_{i1} & -N_{ii} \end{bmatrix}, \quad \dot{\xi}_i(t) = \begin{bmatrix} x(t) \\ x(t - \tau) \end{bmatrix}, \]
\[ Z_i = \begin{bmatrix} N_{11}^T & N_{12}^T \\ N_{21}^T & N_{22}^T \end{bmatrix} \]

(25)

**Lemma 2** [21]. Consider a matrix \( \mathcal{G} \) and the polyhedral set is defined as:
\[ S = \{ \xi; |(\mathcal{K} - \mathcal{G}) \xi(t)| \leq u_0 \} \]

(26)

Then, the dead-zone nonlinearity \( \psi(\mathcal{K} \xi(t)) \) holds the following relation:
\[ \psi(\mathcal{K} \xi(t))^T T_0 [\psi(\mathcal{K} \xi(t)) - G \xi(t)] \leq 0 \]

(27)

## III. LYAPUNOV-KRASOVSKII STABILITY CONDITION FOR SbW SYSTEM WITH ACTUATOR SATURATION

In this section, the anti-windup compensator \( E_c \) for the SbW system in (22) is developed based on the information of the time delay and actuator saturation. The conditions where augmented system stability and bounded control input are proposed to ensure the stability of the closed-loop SbW system via Lyapunov-Krasovskii candidate function. The new anti-windup compensator design is presented in the following theorem:

**Theorem 1** The SbW system in (22) is asymptotically stable if there exist positive definite symmetric matrices \( \mathcal{P} = \mathcal{P}^T > 0, \mathcal{Q} = \mathcal{Q}^T > 0, \) and \( \mathcal{R} = \mathcal{R}^T > 0, \) a positive definite diagonal matrix \( \mathcal{S} \) and a properly dimensioned matrices \( \mathcal{X}, N_i \) for \( i = 1, 2, 3, 4, 5, \) \( \mathcal{V} \) and \( \mathcal{U} \) by fulfilling the following conditions,
\[ \begin{bmatrix} \Omega_{11} & * & * & * & * & * \\ \Omega_{21} & \Omega_{22} & * & * & * & * \\ \Omega_{31} & 0 & \Omega_{33} & * & * & * \\ \Omega_{41} & \Omega_{42} & 0 & \Omega_{44} & * & * \\ \Omega_{51} & \Omega_{52} & 0 & 0 & \Omega_{55} & * \\ \tau \mathcal{N}_1^T & 0 & \tau \mathcal{N}_2^T & 0 & 0 & -\tau \mathcal{R} \end{bmatrix} \]
\[ \begin{bmatrix} X \mathcal{C}_z^T \\ -\mathcal{S} \mathcal{D}_z^T \end{bmatrix} \leq 0 \]

(28)

\[ \begin{bmatrix} \mathcal{P} & \mathcal{X} \mathcal{C}_z^T - \mathcal{V}^T \\ * & \mu \mathcal{U}_0^2 \end{bmatrix} \geq 0 \]

(29)

where

\[ \Omega_{11} = \mathcal{Q} + \mathcal{N}_1 + \mathcal{N}_1^T + \mathcal{X} \mathcal{C}_z \mathcal{A} + \mathcal{A}^T \mathcal{X} \]
\[ \Omega_{21} = \mathcal{P} - \mathcal{X} + \mathcal{X}^T \mathcal{C}_z \mathcal{A} + \mathcal{A}^T \mathcal{X} \]
\[ \Omega_{22} = \tau \mathcal{R} - \mathcal{X} \mathcal{C}_z \mathcal{A} - \mathcal{A}^T \mathcal{X} \]
\[ \Omega_{31} = -\mathcal{N}_1^T + \mathcal{N}_2 
\]
\[ \Omega_{33} = -\mathcal{Q} - \mathcal{N}_2 - \mathcal{N}_2^T \]
\[ \Omega_{41} = -(\mathcal{B} \mathcal{S} + \mathcal{R} \mathcal{U}) + \mathcal{V} \]
\[ \Omega_{42} = -\mathcal{X} \mathcal{C}_z \mathcal{A} + \mathcal{A}^T \mathcal{X} \mathcal{C}_z \mathcal{A} + \mathcal{A}^T \mathcal{A} \]
\[ \Omega_{44} = -2 \mathcal{S} \]
\[ \Omega_{51} = \mathcal{B}_\omega \]
\[ \Omega_{52} = \mathcal{G} \mathcal{B}_\omega \]
\[ \Omega_{55} = -I. \]

(30)

Then, the anti-windup gain can be computed as \( E_c = US^{-1} \).

**Proof.** (Refer to Appendix A)

A schematic representation of the proposed anti-windup scheme is shown in Figure 4. The anti-windup can be introduced to detect and compensate actuator saturation problems of the SbW system with the presence of time delay in control signal.
To calculate the anti-windup compensation gain $E_C$, the proposed Theorem 1 can be formulated into a convex optimization problem. The concept is to minimize scalar $\mu$ to ensure that the closed loop trajectories in (22) remain bounded.

By considering the initial condition is null such that $\xi(0) = 0$, the subsequent LMI can be used to achieve this optimization problem:

$$
\begin{align*}
\min \mu \\
\text{subject to} \quad (28)-(29)
\end{align*}
$$

(31)

IV. SIMULATION RESULTS AND DISCUSSIONS

As indicated in Section 1, a SbW model system is setup with the integration of anti-windup compensation into the system to verify the proposed method. The SbW system is subjected to time delay as the information from the road wheel angle sensors send to the ECU and the reference signal from the steering wheel to the ECU needed time to consider. Therefore, a number of simulation runs are performed on the varying time delays parameters to imitate the real operational of the SbW.

For the simulation experimental setup, the estimated physical parameters are given by [11, 21] as listed in Table I. The nominal operating voltage for DC motor is given in the range of $V_e = -24 \sim 24$ V. Therefore, if the control signals generate control input more than 24 V or less than -24 V, it will cause the actuator to saturate.

Then, matrix $A_p$ and $B_p$ can be obtained as follows:

$$
A_1 = \begin{bmatrix}
-2.4477 & -0.8384 \\
2.5255 & -3.0816 \\
287.3829 & 60.3504 \\
0 & 0 \\
0 & 0 \\
1.2239 & 0 \\
4.0179 & 0 \\
0 & 1 \\
-287.3829 & -20 \\
0 & -282.266 \\
\end{bmatrix}
$$

$$
A_2 = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
\end{bmatrix}
$$

$$
A_p = \begin{bmatrix}
A_1 \\
A_2
\end{bmatrix}
$$

$$
B_p = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & -0.2857 \\
49.2611 & 0
\end{bmatrix}
$$

(32)

The controller that achieves the desired performance control of SbW system is given as

$$
A_c = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix},
B_c = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix}
$$

$$
C_c = \begin{bmatrix}
0.671 & 0.035 & 0.416 & 0.326 & 0.350 \\
0.046 & 0.097 & 0.950 & 0.276 & 0.823 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0
\end{bmatrix} \times 10^3,
$$

$$
D_c = 0.416 \times 10^3
$$

(33)

The system is subjected to disturbances in the form of (15) as follows:

$$
B_w = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
1 & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix},
C_z = \begin{bmatrix}
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix},
D_z = 0
$$

(34)

The optimization problem with objective function (31), which based on the result in Theorem 1 is solved by using Yalmip/Sedumi [22, 23] and the anti-windup compensator $E_c$ can be obtained as:

$$
E_c = \begin{bmatrix}
-0.001 & -0.097 \\
-0.006 & -0.823 \\
0.473 & 5.060 \\
5.043 & 1.386 \\
3.500 & 1.966
\end{bmatrix} \times 10^3
$$

(35)

with tuning parameter $\zeta = 0.7$ and time delay $\tau = 60$ ms.
In order to validate the results of the proposed anti-windup compensation, a controller without anti-windup scheme, which is a conventional state feedback controller will be use as benchmark. This baseline controller is designed without explicitly considering time delays and actuator saturation using pole placement method [24]. Through this method, a controller gain matrix \( K \) that place the poles of the system can be chosen to force the system to have closed-loop poles at the desired locations, hence, the system can be stabilize. By using pole placement method, the gains of state feedback controller can be determined as:

\[
K = 1 \times 10^3 \begin{bmatrix} 0 & -1.0061 \\ 0 & 0.9304 \\ -0.0006 & 0.0898 \\ -0.0057 & -0.0859 \end{bmatrix}^T
\]  

(36)

To investigate the efficiency of the compensation scheme, the energy consumed by DC motor is calculated by the following equation:

\[
E = \int_{0}^{t} |V_o i_r| \, dt
\]  

(37)

where the lower value of \( E \) indicates better energy consumption.

The tracking performance of the SbW system under various case studies is evaluating using integral of the absolute of the error (IAE) index function [11], which is defined as the following equation.

\[
IAE = \int_{0}^{t} |r_s(t) - c_s(t)| \, dt
\]  

(38)

where \( r_s(t) \) is reference signal or desired steering wheel angle input of driving maneuver and \( c_s(t) \) is the parameter that need to measure their performance. For this case, \( c_s(t) \) is the road wheel angle \( \theta_r \). A better tracking control system performance is specified by the lower IAE value.

The simulation is performed until time \( t = 20 \) s. The desired steering angle \( \theta_r \) is initially at \(-1\) rad for \( 5 \) s and changes to \( 1 \) rad for the next \( 5 \) s. This cycle is repeated twice throughout the simulation studies. The control objective of this system is to drive the DC motor at steering rack in such a way the road wheel angle \( \theta_r \) tracking the desired steering angle \( \theta_r \) with less overshoot and minimal oscillation even under long delays bound. The results and analysis can be reviewed in four different cases, which reflect various conditions when the system is in operation with and without anti-windup controller.

**Case 1:** The time delay \( \tau \) set to 0 ms.
**Case 2:** The time delay \( \tau \) set to 60 ms.
**Case 3:** Snowy road condition.
**Case 4:** Double bumps road disturbance.

For simulation studies case 1, 2 and 4, the dry road condition is considered.

The comparative results of the first case, that is under no time delay condition for the system with and without anti-windup controller is depicted in Figure 5. It is found that the DC motor actuator for the SbW system without anti-windup controller is under saturation condition due to the controller has injected a huge control input motor voltage \( V_o \) as much as 435.3 V as shown in Figure 5 (a). While for the proposed anti-windup controller, the control input produced bounded control signal at maximum magnitude of 24 V to the DC motor. Figure 5 (b) show that the controller without anti-windup has driven the road wheel angle \( \theta_r \) to track the reference signal \( \theta_s \) with very high overshoot of 51% in this case, whereas the proposed anti-windup controller driven the road wheel angle with 9.3% overshoot. The IAE of road wheel angle errors for SbW system with and without anti-windup controller are \( IAE(\theta_r)_{\text{with AW}} = 0.02004 \) and \( IAE(\theta_r)_{\text{without AW}} = 0.3087 \), correspondingly. The energy that is consumed by the motor actuator within \( t = 20 \) s are 597.4 Joule and 3329 Joule for the SbW system with the proposed anti-windup and without anti-windup controller respectively, as shown in Figure 5 (c). The energy consumption by the controller without anti-windup is huge and inefficient due to high voltage of control input is needed to produce the desired performance. The vehicle-body sideslip angle response indicates the vehicle stability which the passengers of the vehicle can experience the driving comfort. It can be observed that both controllers preserved vehicle stability as shown in Figure 5 (d). The maximum deflection sideslip angle is occurs at \( t = 10.5 \) s where \( \beta(\text{max})_{\text{with AW}} = -0.291 \) rad and \( \beta(\text{max})_{\text{without AW}} = -0.361 \) rad respectively.

![Control input without anti-windup controller](image1)

(a) Control input motor voltage \( V_o \) of SbW system.
generated within the nominal operating voltage of ±24 V with the proposed anti-windup controller irrespective of the time delay occurrence. In addition, as shown in Figure 6 (b), the vehicle steering tracking performance of the SbW system without anti-windup controller start to deteriorate as the overshoot of the road wheel angle $\theta_r$ increases to 103.3%. The proposed anti-windup controller on the other hand performs better with fewer amount of overshoot that is 35.2%. The IAE value for both system are $IAE(\theta_r)_{with\ AW} = 0.07616$ and $IAE(\theta_r)_{without\ AW} = 0.9157$. Furthermore, Figure 6 (c) shows the resulted motor energy consumption which is $E_{with\ AW} = 642.3$ Joule for SbW with anti-windup controller and $E_{without\ AW} = 3283$ Joule for SbW without anti-windup controller. As denoted in Figure 6 (d), the stability of vehicle dynamics eventually effected for the SbW system without anti-windup controller as the oscillations occurs and increase the maximum deflection sideslip angle to $\beta(\text{max})_{without\ AW} = -0.433$ rad. In this situation, the passengers of the vehicle may experience discomfort while driving. Conversely, the vehicle dynamics response with the proposed anti-windup controller remain stable with little oscillation effect and has maximum deflection sideslip angle at $\beta(\text{max})_{with\ AW} = -0.300$ rad.

Then, the simulation is carried out with the time delay 60 ms introduced into the system, the SbW system without anti-windup controller produced more chattering signal to DC motor as specified in Figure 6 (a) which can cause excessive heating of the motor coil and decreasing the life span of the actuator. While it is shown that stable control input and
The simulation studies continue to further validate the priority of the proposed controller with different road friction condition. Note that the previous simulation cases are employed on the dry road, and for this case study, the SbW system is applied on snowy road condition. The steering angle tracking performance is shown in Figure 7 (a) when the SbW system is in operation with and without anti-windup controller. It can be seen from Figure 7 (a) that the vehicle steering performance of controller with the proposed anti-windup works better as compared to controller without anti-windup with measurement of overshoot 40.2% and 76.2% correspondingly. The IAE value for both system are noted as $IAE(\theta_r)_{\text{with AW}} = 0.06928$ and $IAE(\theta_r)_{\text{without AW}} = 0.4594$. As expected, when the road surface changed from dry to snow it will effect the steering tracking control system. Nevertheless, satisfactory result is achieved. Moreover, the vehicle-body sideslip angle response of the vehicle dynamics has changed compared to previous aforementioned simulation studies as shown in Figure 7 (b). The vehicle dynamics response is almost the same in the first 10 s for the two controllers, and relatively large sideslip angle deflection happens at $t = 15.5$ s with $\beta(\text{max})_{\text{with AW}} = 0.65$ rad and $\beta(\text{max})_{\text{without AW}} = 0.83$ rad respectively. In this situation, the proposed anti-windup controller conserves better stability and comfort driving experience.

Next, in order to assess the robustness of the proposed control method, the SbW vehicle bumped down the dry road condition and triangle steering maneuver has implemented to easily observed the system response. The triangle steering maneuver has amplitude 1 rad at $t = 5$ s and, a double bumps road disturbance function is modified in the form of [25]:

$$d(t) = \begin{cases} 1 + \cos(0.5\pi t) & \text{if } 10 \leq t \leq 14 \\ 1 + \cos(0.5\pi t) & \text{if } 18 \leq t \leq 12 \\ 0 & \text{otherwise} \end{cases}$$

(39)

Figure 8 (a) shows the road wheel angle response of the SbW system performs under bumps road disturbance. It can be perceived that both controllers fairly good to track the triangle steering reference signal in the first 9 s. After that, there are some deviations in the road wheel angle when going through the bumps disturbance and the deviation angle of the proposed anti-windup controller is smaller compared with that of the controller without anti-windup. Their performance measurement of IAE value are recorded as $IAE(\theta_r)_{\text{with AW}} = 0.1563$ and $IAE(\theta_r)_{\text{without AW}} = 0.2554$. The vehicle dynamics result shown in Figure 8 (b) is different as those of previous maneuver in response to the bumps road disturbance.
In this case, the maximum sideslip angle deflection occurs at $t = 21.5$ s with $\beta(\text{max})_{\text{with AW}} = 0.1227$ rad for the SbW with the proposed anti-windup controller. For the comparison purpose, the sideslip angle response of the SbW without anti-windup controller is a bit worse than the one of the proposed control with $\beta(\text{max})_{\text{without AW}} = 0.1437$ rad. In this condition, both controller does not cause the vehicle instability and maintain the overall control system performance.

Table II shows the summarize performance comparison of both SbW system with and without anti-windup compensation. From the results, the SbW system in operation with the proposed anti-windup compensation not only solves actuator saturation problem but also promotes a better performance in terms of smaller control input motor voltage $V_r$, less percent overshoot of road wheel angle $\theta_r$, smaller steering tracking error $\text{IAE} (\theta_r)$, better energy $E$ consumption and smaller sideslip angle $\beta$ deflection for all case studies.

Table II THE PERFORMANCE COMPARISON BETWEEN SbW SYSTEM WITH AND WITHOUT ANTI-WINDUP COMPENSATION.

<table>
<thead>
<tr>
<th></th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\tau = 0$ ms</td>
<td>$\tau = 60$ ms</td>
<td>Snowy road condition</td>
<td>Bumps road disturbance</td>
</tr>
<tr>
<td></td>
<td>With anti-windup</td>
<td>Without anti-windup</td>
<td>With anti-windup</td>
<td>Without anti-windup</td>
</tr>
<tr>
<td>Control input $V_r$ (V)</td>
<td>$\pm24.0$</td>
<td>$343.3$</td>
<td>$\pm24.0$</td>
<td>$206.0$</td>
</tr>
<tr>
<td>Percent overshoot (%)</td>
<td>$9.3$</td>
<td>$51.0$</td>
<td>$35.2$</td>
<td>$103.3$</td>
</tr>
<tr>
<td>$\text{IAE}$</td>
<td>$0.02004$</td>
<td>$0.3087$</td>
<td>$0.07616$</td>
<td>$0.361$</td>
</tr>
<tr>
<td>Motor energy $E$ (Joule)</td>
<td>$597.4$</td>
<td>$3329.0$</td>
<td>$642.3$</td>
<td>$3283.0$</td>
</tr>
<tr>
<td>Sideslip angle $\beta$ (rad)</td>
<td>$-0.291$</td>
<td>$-0.361$</td>
<td>$-0.300$</td>
<td>$-0.433$</td>
</tr>
</tbody>
</table>

V. CONCLUSION

This paper discussed a new anti-windup scheme designed to ensure stability of SbW system in the presence of time delays and actuator saturation. Coupled with controller used in the SbW systems, an anti-windup compensator designs begin with the formation of augmented state matrices and then, the stabilization is expressed in the form of LMIs which is derived based on Lyapunov-Krasovskii candidate function. The effectiveness of this approach is demonstrated by applying the scheme to the SbW system. Evidence from the simulation works, the proposed anti-windup scheme provides stability and consistency steering performance during control saturation in the presence of time delays as compared to controller without anti-windup scheme. For future works, the vehicle model with associated nonlinearities will be considered and a new anti-windup SbW compensation will be proposed.

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REFERENCES


APPENDIX A

Proof Theorem 1.

Take the following Lyapunov-Krasovskii candidate function:

\[
V(t) = \xi^T(t) P \xi(t) + \int_0^t \xi^T(s) Q \xi(s) ds \quad \text{for} \quad t \geq 0
\]

A.1

where matrices \( P = P^T > 0, Q = Q^T > 0 \) and \( R = R^T > 0 \) are positive symmetric.

Time differentiating the Lyapunov-Krasovskii candidate function (A.1) along trajectory of system in (22) gives

\[
\dot{V}(t) = \xi^T(t) P \dot{\xi}(t) + \dot{\xi}^T(t) Q \xi(t) - \xi^T(t - \tau) Q \xi(t - \tau) + \rho \xi^T(t) R \xi(t) - \int_{t-\tau}^{t} \dot{\xi}^T(s) R \dot{\xi}(s) ds
\]

(A.2)

Using lemma 1, the integral term in (A.2) can be bounded as follows:

\[
- \int_{t-\tau}^{t} \dot{\xi}^T(s) R \dot{\xi}(s) ds \leq 2 \left[ \xi^T(t) - \xi^T(t - \tau) \right] \left[ \begin{array}{c} N_1 \\ -N_2 \end{array} \right] \left[ \begin{array}{c} \xi(t) \\ \xi(t - \tau) \end{array} \right] + \tau \xi^T(t) \xi(t - \tau) + \tau \xi^T(t) \left[ \begin{array}{c} N_1 \\ -N_2 \end{array} \right] R \left[ \begin{array}{c} N_1 \\ -N_2 \end{array} \right] \left[ \begin{array}{c} \xi(t) \\ \xi(t - \tau) \end{array} \right]
\]

(A.3)

The system in (22) is added into the convexity condition in Theorem 1 through the free weighting matrix method as proposed in [26]. Hence, it can be deduced that:

\[
2 \left[ \xi^T(t) T_1 + \dot{\xi}^T(t) T_2 \right] \left[ \begin{array}{c} \xi(t) \\ \xi(t - \tau) \end{array} \right] + \left( \begin{array}{c} A \xi(t) \\ B \phi \xi(t) + B_0 \omega(t) \end{array} \right) = 0
\]

(A.4)

Substituting (A.3) into (A.2) and adding the free weighting matrix term (A.4) and lemma 2 on the \( \dot{V}(t) \), the time differentiation of the Lyapunov-Krasovskii candidate function becomes,

\[
\dot{V}(t) = \xi^T(t) P \xi(t) + \xi^T(t) P \xi(t) + \xi^T(t) Q \xi(t) - \xi^T(t - \tau) Q \xi(t - \tau) + \rho \xi^T(t) R \xi(t)
\]
Then, the objective function is defined for a prescribed scalar $y$ as follows:

$$J(t) \leq V(t) - \omega^T(t)\omega(t) + \frac{1}{y} z^T(t)z(t) \quad (A.6)$$

Therefore,

$$J(t) \leq \xi^T(t)P_1\xi(t) + \xi^T(t)P_1\xi(t) + \xi^T(t)Q\xi(t) - \xi^T(t)Q\xi(t) + \tau \xi^T(t)R\xi(t) + \frac{1}{y} z^T(t)\psi^T(\mathcal{K}\xi(t))$$

$$\times \left[\begin{array}{c}
\xi(t) \\
\xi(t) \\
\end{array}\right] + 2(\xi^T(t)T_1 + \xi^T(t)T_2)$$

$$(\xi(t) + \eta(t) - A\xi(t) - (B + RE_c)\psi\mathcal{K}\xi(t))$$

$$+ B_{wo}(\omega(t)) - 2\psi^T(\mathcal{K}\xi(t))T_0\psi(\mathcal{K}\xi(t))$$

$$- G\xi(t) - \omega^T(t)\omega(t) + \frac{1}{y} z^T(t)\psi^T(\mathcal{K}\xi(t))$$

$$\times \left[\begin{array}{c}
C_z^T \\
-D_z^T \\
\end{array}\right] \left[\begin{array}{c}
\psi(\mathcal{K}\xi(t)) \\
\end{array}\right] \quad (A.7)$$

By rearranging the terms in (A.7) and reformulating the equation into a matrix form, the subsequent inequality holds:

$$J(t) \leq \Xi^T(t)\pounds(t) + \tau \xi^T(t)\xi(t)$$

$$\times \left[\begin{array}{c}
N_1 \\
N_2 \\
\end{array}\right] R^{-1}[N_1^T N_2^T]$$

$$+ \xi^T(t)\psi^T(\mathcal{K}\xi(t))$$

$$\times \left[\begin{array}{c}
C_z^T \\
-D_z^T \\
\end{array}\right] \psi(\mathcal{K}\xi(t)) \quad (A.8)$$

where

$$\Xi(t) = [\xi^T(t) \xi^T(t) \xi^T(t) \psi^T(\mathcal{K}\xi(t)) \omega^T(t)]$$

$$\phi = [\phi_{11} \phi_{22} \phi_{33} \phi_{44} \phi_{55}]$$

$$\phi_{11} = Q + N_1^T N_1 + T_1^T S_1 T_1$$

$$\phi_{22} = \tau R - T_2 - T_2^T$$

$$\phi_{33} = -N_1^T + N_2$$

$$\phi_{44} = -Q - N_3 - N_2^T$$

$$\phi_{55} = -2T_0$$

By Schur complement, the following condition is obtained where $J(t)$ is negative definite. Thus, the SbW system in (22) is asymptotically stable, such that:

$$\Xi^T(t)\Xi(t) - \phi \leq 0 \quad (A.9)$$

Through a standard change of variables, the conditions in Theorem 1 can be solved by changing the original variables $P, N_1, N_2, Q$ and $R$ to new LMI variables $\tilde{P}, \tilde{N}_1, \tilde{N}_2, \tilde{Q}$ and $\tilde{R}$. Define $T_2 = \xi T_1$ where $\xi$ is a scalar tuning parameter. Multiplying $\Delta^T$ on the left-hand side and $\Delta$ on the right-hand side of (A.10) where $\Delta$ is defined as $\Delta = \text{diag} \{X, X, X, I, X, I\}$, then the following equations are defined:

$$X = T_1^{-1}, \tilde{P} = X^T P X, \tilde{N}_i = X^T N_i X \quad \text{for } i = 1, 2$$

$$\tilde{Q} = X^T Q X, \tilde{R} = X^T R X, U = E_c T_0^{-1}, S = T_0^{-1}$$

$$V = G X^T$$

Therefore, the LMI condition in (28) is obtained and the desired anti-windup compensator can be calculated as $E_c = US^{-1}$. From (26) and (27), the following inequality is obtained as proposed in [20].

$$\left[\begin{array}{c}
P - \kappa^T - G^T \mu \mu^T \\
\end{array}\right] \geq 0 \quad (A.12)$$

Define that $\Lambda = \text{diag} \{X, I\}$. Then, multiplying $\Lambda^T$ on the left-hand side and $\Lambda$ on the right-hand side of (A.12), LMI (29) of Theorem 1 is obtained. This completes the proof of the theorem. □