Handling Improvement of a Four-Wheeled Vehicle Using a New Control System

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Abstract-- In this paper to increase the maneuverability and lateral stability of a vehicle a new control system is proposed. First a fourteen-degrees-of-freedom nonlinear dynamic model of a four-wheeled vehicle is developed. Then the vehicle model is validated using real test data and ADAMS CAR software during different maneuvers. Next to improve the vehicle dynamic performance, a new control system designed based on a simplified dynamic model. Also, the control system performance is evaluated at different velocities. Simulation results show that the controller improves the vehicle’s handling, especially during severe slalom maneuver in which intense instability occurs. Moreover, the proposed control system is robust against variations in the parameters and in the velocity of the vehicle.

Index Term-- Lateral Stability, control system, nonlinear dynamic model, dynamic performance.

I. INTRODUCTION

The use of control systems to improve the lateral stability and the maneuverability increment of the vehicle to reduce the number of road accidents has been considered considerably. The more researches show that the vehicles’ lateral motions are controlled by active steering control systems and active braking control systems as well as Active roll control system. It can be note that the longitudinal motion is controlled by accelerator/brake pedals. Moreover, Active steering control systems is considered as the most effective methods to affect the vehicle maneuverability as they are traditionally the primary vehicle input. Steering is also the most direct form as it is the only input that directly controls the amount of generated lateral force [1-7]. Given the road condition and tire normal force, the lateral force produced from the tire is a function of the tire slip ratio and slip angle. Changing the steering angle will affect the vehicle lateral dynamics. Many studies have been done using active control systems to increase the vehicle maneuverability [8,9]. Some studies have emphasized only the development of the control system for lateral stability improvement [10]. Other researchers proposed linear quadratic control system to eliminate the error between the actual state variable of the vehicle and its desired value [11,12,13]. Also, many investigations have been done about controlling vehicle slip ratio to generate sufficient lateral forces and longitudinal forces [14].

Kim et al. [15] proposed an active steering control system to achieve improvement of vehicle stability at cornering conditions. Many methods have been studied and actively developed to improve a four-wheeled vehicle’s lateral stability actively. Guvenc et al. [16] proposed an active control system in order to improve the handling and the lateral stability of the four-wheeled vehicle. The control system is designed with the application of model matching control theory. They evaluate the performance of the active steering control system during standard maneuvers at different conditions. Nam et al. [17] in order to increase the lateral stability and maneuverability of a four-wheeled vehicle, a new control system is designed based on robust lateral tire force control. Then, the vehicle performance is evaluated during cornering at various velocities. Moreover, a robust control method with a disturbance observer has been utilized to reduce the error between the actual state of the vehicle and its desired value. Nashar et al. [18] uses LQR control method to develop an active steering control system. This control system neglects the important nonlinearities due to suspension and tire characteristics that would be observed in a real vehicle at the high lateral accelerations discussed.

The organization of this paper is as follows. In the second section, the vehicle dynamic modeling is presented. Then, the nonlinear dynamic model is validated using real test data and ADAMS CAR software in standard maneuver. Next, in order to improve the dynamic performance of the vehicle, a new control system is designed based on a simplified dynamic model. In fifth section, the control system performance is evaluated at different velocities. Next, the results of simulations in different conditions are presented. Finally, Conclusions are provided.

II. VEHICLE DYNAMIC MODELING

Fig. 1 shows a fourteen-degrees-of-freedom nonlinear dynamic model used in this paper. This 14-DOF model includes the longitudinal, lateral, vertical, yaw, roll and pitch motions of the vehicle [13]. Each of the wheels has translational motion in vertical direction and wheel spin about lateral direction. The front wheels can steer about the z-axis. In the development of the vehicle model, the following assumptions were made:

1. The steering angles δ of both front wheels are considered identical.
2. The effect of unsprung mass is ignored in the vehicle’s pitch and roll motions.
3. The tire and suspension remain normal to the ground during vehicle maneuvers.
4. The center of roll and pitch motion are placed on the vehicle’s center of gravity.

A. Equations of motion

The longitudinal, lateral and vertical motions are defined by

\[
M_x (\ddot{v}_x + \dot{v}_y \dot{\psi} - v_y \dot{\psi}) = F X_{Ff} + F X_{Ff} + F X_{Ff} + F X_{Ff} \tag{1}
\]

\[
M_y (\ddot{v}_y + \dot{v}_y \dot{\psi} - v_x \dot{\psi}) = F Y_{Ff} + F Y_{Ff} + F Y_{Ff} + F Y_{Ff} \tag{2}
\]

\[
M_\theta (\ddot{\psi} + \dot{\psi}^2) = F Z_{Ff} + F Z_{Ff} + F Z_{Ff} + F Z_{Ff} \tag{3}
\]

Where \(v_x\) is the vehicle’s longitudinal velocity, \(v_y\) is the vehicle’s lateral velocity, \(\dot{\psi}\) is the yaw rate, \(\theta\) is the pitch rate and \(\phi\) is the roll rate. Also, \(F_{Ff}\) and \(F_{Yf}\) and \(F_{Zf}\) and \(F_{rf}\), \(i = f, l, r, l, r, r, r\), are the tire forces in the \(x, y\) and \(z\) directions respectively, which can be related to the tire’s tractive force \(F_{Xf}\) and the tire’s lateral force \(F_{Yf}\) as

\[
F_{Xf} = F_{Xf} \cos(\delta_f) - F_{Yf} \sin(\delta_f),
\]

\[\text{for } (i = f, r, (k = l, r))\]

\[
F_{Yf} = F_{Yf} \cos(\delta_f) + F_{Xf} \sin(\delta_f),
\]

\[\text{for } (i = f, r, (k = l, r))\]

The roll, pitch and yaw motions are defined by

\[
M_x = L_{xx} \ddot{\psi} - (l_{yy} - l_{xx}) \dot{\psi} \dot{\psi} = (F Z_{Ff} + F Z_{Ff}) T_{Yf} / 2 + (F Z_{Ff} + F Z_{Ff}) T_{Yf} / 2 - (F Y_{Ff} + F Y_{Ff} + F Y_{Ff} + F Y_{Ff}) \tag{5}
\]

\[
(h_{cg} - h_{cr}) \dot{\dot{\psi}} \]

\[
M_y = l_{yy} \ddot{\psi} - (l_{xx} - l_{yy}) \dot{\psi} \dot{\psi} = (F Z_{Ff} + F Z_{Ff}) L_{Yf} - (F Z_{Ff} + F Z_{Ff}) L_{Yf} + (F Y_{Ff} + F Y_{Ff} + F Y_{Ff} + F Y_{Ff} + F Y_{Ff} + F Y_{Ff}) \tag{6}
\]

\[
(h_{cg} - h_{cr}) \dot{\dot{\psi}} \]

\[
M_\theta = l_{zz} \ddot{\psi} - (l_{xx} - l_{yy}) \dot{\psi} \dot{\psi} = (F Y_{Ff} + F Y_{Ff}) T_{Yf} / 2 + (F Y_{Ff} + F Y_{Ff}) T_{Yf} / 2 + (F Y_{Ff} + F Y_{Ff}) L_{Yf} - (F Y_{Ff} + F Y_{Ff}) \tag{7}
\]

The equations of motion for the suspension system are derived as follows.

\[
F Z_{Ff} = k s_{fr} \left(z_{ufr} - z_{fr}\right) + c s_{fr} \left(\dot{z}_{ufr} - \dot{z}_{fr}\right) \tag{11}
\]

\[
F Z_{f1} = k s_{fr} \left(z_{uf1} - z_{f1}\right) + c s_{fr} \left(\dot{z}_{uf1} - \dot{z}_{f1}\right)
\]

\[
F Z_{r1} = k s_{fr} \left(z_{ur1} - z_{r1}\right) + c s_{fr} \left(\dot{z}_{ur1} - \dot{z}_{r1}\right)
\]

The tire forces are defined as:

\[
F Z_{f1} = k s_{fr} \left(z_{uf1} - z_{uf}\right) + c s_{fr} \left(\dot{z}_{uf1} - \dot{z}_{uf}\right) \tag{12}
\]

\[
F Z_{f1} = k s_{fr} \left(z_{uf1} - z_{uf}\right) + c s_{fr} \left(\dot{z}_{uf1} - \dot{z}_{uf}\right)
\]
\[ F_{z_{trr}} = k_{trr}(z_{sr} - z_{ur}) + c_{trr}(\dot{z}_{sr} - \dot{z}_{ur}) \]
\[ F_{z_{trl}} = k_{trl}(z_{sr} - z_{ur}) + c_{trl}(\dot{z}_{sr} - \dot{z}_{ur}) \]

and
\[ z_{fr} = z + \frac{T_f}{2} \phi - L_f \theta \]
\[ z_{fl} = z - \frac{T_f}{2} \phi - L_f \theta \]
\[ z_{rr} = z + \frac{T_r}{2} \phi + L_r \theta \]
\[ z_{rl} = z - \frac{T_r}{2} \phi + L_r \theta \]

The normal load applied to each tire is a function of both the vehicle’s static load and dynamic load transfer. Thus, the normal load equation for each wheel can be represented as:

\[ F_{n_{fr}} = \left( m_{u_{fr}} + \frac{m_{sL}}{2l} \right) g - F_{z_{fr}} \]
\[ F_{n_{fl}} = \left( m_{u_{fl}} + \frac{m_{sL}}{2l} \right) g - F_{z_{fl}} \]
\[ F_{n_{rr}} = \left( m_{u_{rr}} + \frac{m_{sR}}{2l} \right) g - F_{z_{rr}} \]
\[ F_{n_{rl}} = \left( m_{u_{rl}} + \frac{m_{sR}}{2l} \right) g - F_{z_{trl}} \]

**B. Tire dynamics**

One of the most importance stages to simulate the vehicle dynamics is the tire modelling. In the linear model, due to the direct relation between the lateral force and the tyre side slip angle, the tire normal forces are ignored. In this condition, the tire potential is disregarded for slip prevention and its saturation. As far as the linear tire model crosses the linear zone boundaries at high velocities, as well as high side slip angles, a nonlinear tire model is considered. The traction and side forces acting on the vehicle are generated at the contact path between tire and road. The traction forces are generated for acceleration or braking and the side forces are necessary to adjust the direction of the vehicle. In this article, the Magic Formula model is used to simulate the tire forces [20].

\[ F_x = Fx0.\cos((Rcx1.\tan(Bx0.\alpha)) - (Rex1 + Rex2.\df z). (Bx0.\alpha) - \tan(Bx0.\alpha)))/ \]
\[ \cos((Rcx1.\tan(Bx0.\Rxh) - (Rex1 + Rex2.\df z). (Bx0.\Rxh) - \atan(Bx0.\Rxh)) \]

\[ F_y = Gy0.\cos((Rcy1.\atan(f.u.S) - (Rey1 + Rey2.\df z). (f.u.S) - \atan(f.u.S)))/ \]
\[ \cos((Rcy1.\atan(f.u.\Ryh + Rey2.\df z)) - (Rey1 + Rey2.\df z) - \atan(fu.(\Ryh + Rey2.\df z))). \]

**C. Wheel dynamics**

The rotational motion of each wheel is represented as follows:

\[ l_w \omega = -F_x. R_w + T_i \]

Where \( \omega \) is the rotational velocity, \( l_w \) is the moment inertia of the wheel, \( F_x \) is the longitudinal force, \( R_w \) is the tire rolling radius and \( T \) is the input drive.

**III. VALIDATION OF FOUR-WHEELED VEHICLE MODEL**

In this paper, in order to validate the vehicle dynamic model ADAMS CAR software is utilized during a standard maneuver. As can be seen from Figs, the lateral acceleration and the yaw rate were well matched with the real test data. Also, in order to investigate the vehicle model ADAMS CAR software is utilized during a standard maneuver. In this maneuver, the vehicle runs at initial velocity of 50 \( km/h \) on a dry road. The steering input is shown in Fig. 5. Moreover, although the yaw rate curve is in fairly good agreement with the ADAMS CAR results, some deviations are observed between the two models.
IV. CONTROL SYSTEM DESIGN

In this section to design an active steering control system a simplified two-degrees-of-freedom dynamic model is used. In the control system the yaw rate of the vehicle is considered as state variable. The main objective of the controller is to eliminate the error between the actual yaw rate and its desired value. The two-degrees-of-freedom linear bicycle model with kinematic quantities and lateral tire forces are taken from our previous paper [13]. This model describes the lateral motions of the vehicle. Also, this simplified dynamic model is a useful tool for investigating the key features of vehicle handling performance. For such a model, the following set of assumptions is made to further idealize the vehicle motions:

- The left and right wheels on the same axle are laterally lumped into one in the centerline.
- The vehicle is running at a constant velocity $v_x$. Also, the longitudinal forces will not be examined.
- The vehicle structure, including the suspension system, is rigid.
- Both longitudinal and lateral load transfers are neglected.

The vehicle can be viewed as consisting of a planar motion described by two variables: the lateral velocity $v_y$ and the yaw rate $r$. The equations of motion of the bicycle model can then be expressed as follows by directly applying Newton’s Second Law:

$$m(v_y + v_x r) = F_{yr} + F_{yf} \cos(\delta)$$  \hspace{1cm} (17)

$$I_{zz} \ddot{r} = F_{yf} l_1 \cos(\delta) - F_{yr} l_2$$  \hspace{1cm} (18)
Where \( I_{zz} \) is the moment of inertia of the vehicle about its yaw axis, \( m \) is the vehicle mass, \( l_1 \) and \( l_2 \) are distances of the front and rear axles to the center of gravity and \( F_{yf} \) and \( F_{yr} \) are lateral tire forces for front and rear axles.

In line with the small angle assumptions, one can have \( \cos \delta = 1 \) and the lateral tire forces of both axles can be expressed as the product of cornering stiffness \( C_i \) and tire slip angle \( \alpha_i \):

\[
F_{yf} = -C_{af}\alpha_f
\]
\[
F_{yr} = -C_{ar}\alpha_r
\]  

(19)

It should be noted that the above two equations are written in terms of axles, which means the cornering stiffness is that of the corresponding axle rather than that of the single tire. In addition, the front and rear tire slip angles can be approximated as:

\[
\alpha_f = \frac{v_y + l_1 r}{v_x} - \delta
\]
\[
\alpha_r = \frac{v_y - l_2 r}{v_x}
\]  

(20)

Substituting Eqs. (19) and (20) into Eqs. (17) and (18) to give:

\[
m(v_y + v_x r) = -\left(\frac{c_{af} + c_{ar}}{u}\right) v_y - \left(\frac{l_1 c_{af} - l_2 c_{ar}}{u}\right) r + c_{af} \delta
\]  

(21)

\[
I_{zz} \dot{r} = -\left(\frac{l_1 c_{af} - l_2 c_{ar}}{u}\right) v_y - \left(\frac{c_{af} l_1^2 + c_{ar} l_2^2}{u}\right) r + l_1 c_{af} \delta
\]  

(22)

Rearranging to give the following state-space representation:

\[
\dot{x} = Ax + Bu
\]  

(23)

where the state vector \( x \), the input vector \( u \), the system matrix \( A \) and the input matrix \( B \) are defined by:

\[
x = \begin{pmatrix} v_y \\ r \end{pmatrix}, \quad A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}
\]  

(24)

\[
a_{11} = \frac{c_r + \epsilon_f}{mu}, \quad a_{12} = \frac{(l_2 c_r - l_1 \epsilon_f)}{mu} - v_x
\]
\[
a_{21} = \frac{(l_2 c_r - l_1 \epsilon_f)}{l_{zz} u}, \quad a_{22} = -\frac{(l_2 c_r + l_1 \epsilon_f)}{l_{zz} u}
\]  

(25)

\[
b_1 = \frac{c_f}{m}, \quad b_2 = \frac{l_1 \epsilon_f}{l_{zz}}
\]

For the yaw rate tracking task, the difference between the actual yaw rate and its desired value describes the tracking error:

\[
e = r - r_d
\]  

(26)

By derivation from the Eq. (26)

\[
\dot{e} = \dot{r} - \dot{r}_d
\]  

(27)

The sliding surface is then selected as:

\[
s = e
\]  

(28)

Here, the sliding surface can be interpreted as the surface of the yaw rate error between the vehicle and the reference model. As \( s \) goes to zero, the AFS can track the reference yaw rate perfectly. The solution \( s = 0 \) is rigorous but difficult to use for controller design. A better approach for controller design is to introduce the so-called equivalent control method for defining the system behavior in the course of sliding mode, i.e., the sliding motion can be viewed as an average of the system dynamics on both sides of the sliding surface. The equivalent control is defined by the following equation:

\[
\dot{s} = 0
\]  

(29)

\[
\dot{s} = \dot{r} - \dot{r}_d
\]  

From the Eq. (23) we have

\[
\dot{r} = a_{21} v_y + a_{22} r + b_2 u
\]  

(30)

Substituting the Eq. (30) into Eq. (29) gives:

\[
a_{21} v_y + a_{22} r + b_2 u - \dot{r}_d = 0
\]  

(31)

\[
\dot{u} = \tilde{b}_2^{-1} (\tilde{a}_2 v_y + \tilde{a}_2 r - \dot{r}_d)
\]

The control input is

\[
u = \dot{u} - k \text{sgn}(s)
\]  

(32)

where \( k \) is a positive parameter to be tuned in the controller design and \( \text{sgn}(s) \) is the sign function, respectively.

The diagram of the control system is shown in Fig. 7.

![Diagram of control system](image)

**Fig. 7.** The diagram of control system.

V. SIMULATION RESULTS

A. Lane change maneuver

In this maneuver, the vehicle runs at initial velocity of 80 km/h on a dry road with 0.7 friction coefficient and the steering input shown in Fig. 8:
The simulation results are shown for the most important dynamic responses of the vehicle in Figs. 9 to 11. As can be seen from Fig. 9 the uncontrolled vehicle yaw rate deviates from its desired value for uncontrolled condition. In this state, the vehicle lateral velocity increases considerably. In the controlled condition, the active steering control system makes the yaw rate of the vehicle tracks its desired value. In the controlled case, the vehicle side slip is limited in a narrow band. However, the vehicle lateral stability increases. According to Fig. 10(b), the vehicle without control system is severely unstable and leaves its path. However, the tracking of the desired yaw rate leads to the vehicle motion in its desired trajectory. The control effort is shown in Fig. 11(a) has a smooth form. As can be seen from Fig. 11(b), the controller is able to reduce the lateral acceleration considerably in comparison with uncontrolled condition.

Fig. 8. The steering angle.

Fig. 9. Yaw rate

(a)

Lateral Velocity

(b)

Vehicle Path

Fig. 10. (a) lateral velocity, (b) vehicle trajectory.
B. Slalom maneuver

In this maneuver, the vehicle runs on an icy road with a friction coefficient of 0.3 at the constant speed of 100 km/h and the steering input shown in Fig. 12.

The yaw rate, lateral velocity, vehicle path, the lateral acceleration and control effort of the vehicle are illustrated in Figs. 13 and 14. According to Fig. 13, the vehicle yaw rate deviates from its desired value greatly due to yaw instability occurrence. However, the active steering control system is able to make the yaw rate follows its desired value appropriately. Fig. 14(a) shows that the vehicle lateral velocity becomes great at the uncontrolled condition. While using the controller, a considerable reduction at the lateral velocity is observed during steady state and transient maneuver. The vehicle trajectory is presented in Fig. 14(b) at two conditions. For uncontrolled state, the vehicle deviates from its path while for the controlled vehicle no path departure is observed. As can be seen from Fig. 14(c), the proposed control system reduces the lateral acceleration’s peak values significantly for steady state and transient condition.
VI. CONCLUSION

In this paper, to increase the lateral stability and handling improvement of a four-wheeled vehicle an active steering control system is proposed. First, in order to simulate the dynamic model of a four-wheeled vehicle a fourteen-degrees-of-freedom nonlinear dynamic model is developed. Then, the vehicle dynamic model is validated in standard maneuvers. Next, to improve the dynamic performance of the vehicle, a new active steering controller is designed. Therefore, a simplified dynamic model is used to extract the control inputs. Also, sliding mode control method is used to design the controller. Then, the control system performance is evaluated in critical maneuvers. The followings are main conclusions considered here:

- The uncontrolled vehicle subjected to yaw instability in slalom maneuver. In the controlled state, the active steering control system is able to improve vehicle handling at the critical condition.
- The lateral velocity of the vehicle without control system increases for both maneuvers. However, using the controller the vehicle lateral velocity is limited in a narrow band. In this state, the vehicle lateral stability increases.
- Using the controller, a considerable reduction in lateral acceleration is observed, especially in the slalom maneuver.
- The vehicle handling is strengthened by using a controller which is robust against road condition variation and the different vehicle velocity.
APPENDIX

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<td>$C_{sr}$ (KN m s/rad)</td>
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NOMENCLATURE

- $C_{si}$: front/ rear suspension damping constant
- $C_{st}$: Front/ rear tire damping constant
- $h_s$: height of sprung mass
- $I_w$: Wheel moment of inertia
- $I_{xx}$: roll moment of inertia
- $I_{yy}$: pitch moment of inertia
- $I_{zz}$: yaw moment of inertia
- $L_f$: distance of the center of gravity from the front axle
- $L_r$: distance of the center of gravity from the rear axle
- $M_s$: Vehicle sprung mass
- $M_{uf}$: front unsprung mass
- $M_{ur}$: rear unsprung mass
- $M_t$: Vehicle total mass
- $T_f$: front track width
- $T_r$: rear track width
- $\psi$: yaw angle
- $\delta$: steer angle
- $v_x$: longitudinal velocity
- $v_y$: lateral velocity
- $v_z$: vertical velocity
- $F_x$: tire’s longitudinal force
- $F_y$: tire’s lateral force
- $r$: yaw rate
- $c_a$: Cornering stiffness

REFERENCES


