Effect Of The Finite Ground Conductivity On The Lightning-Induced Voltage On Overhead Line

M. Bidi¹, M.H. Latreche¹
¹Electrotechnical Laboratory "LEC", Faculty of Technology, University of Constantine -1-, Algeria
*E-mail: bidi_manel@yahoo.fr

Abstract— The aim of this paper is to investigate the effect of lossy ground conductivity on the lightning induced overvoltage on overhead lines. The ground conductivity plays a role in both the evaluation of the lightning field radiated and the line parameters. Regarding the radiated electromagnetic field, it is shown that the horizontal electric field, the component which is affected by the finite ground conductivity, can be calculated using the Cooray-Rubinstein formula. We also investigate the time-domain representation of field-to-transmission line coupling equations where the frequency-dependence of the ground is taken into account by the convolution integral.

Index Term— EM field, lightning-induced voltage, lossy ground, overhead line.

I. INTRODUCTION

Lightning-induced voltage in overhead lines is an important source of failures. The assumption we discuss in this paper concerns the ground conductivity. Indeed, in most of studies on the subject, the ground has been assumed as a perfectly conducting plane. However, the ground conductivity plays a role in both the evaluation of the lightning radiated field and the line parameters.

The horizontal component of the electric field is more affected by the imperfect ground conductor, it can be calculated using the Cooray-Rubinstein formula and it is used as a source terms in the coupling model (Agrawal). To solve the transmission line coupling equations in the time domain, we use the finite difference time domain (FDTD) technique in MATLAB environment.

II. CALCULATION OF EM FIELD RADIATED BY LIGHTNING RETURN STROKE

The configuration of the system is shown in Fig.1

A. The case of perfectly conducting ground

In that case, the components of the electric and magnetic field at the location p(r,φ,z), as in Fig.1, produced by a short vertical section of infinitesimal channel dz at height z’ carrying a time-varying current i(z’,t) can be computed in the following relations [1], [2]:

\[ dE_c(x, y, t) = \frac{dz'}{4\pi \epsilon_0} \left[ \frac{2(z-z')^2-r^2}{R^3} \right] i(z', \tau = \frac{R}{c}) d\tau + \frac{2(z-z')^2-r^2}{cR^4} \frac{r^2}{c^2} \frac{\partial}{\partial t} i(z', \tau = \frac{R}{c}) \]

\[ dE_h(x, y, t) = \frac{dz'}{4\pi \epsilon_0} \left[ \frac{3r(z-z')}{R^3} \right] i(z', \tau = \frac{R}{c}) d\tau + \frac{3r(z-z')}{cR^4} \frac{r(z-z')}{c^2} \frac{\partial}{\partial t} i(z', \tau = \frac{R}{c}) \]

\[ dB_o = \frac{\mu_0 dz'}{4\pi} \left[ \frac{r}{R^3} i(z', \tau = \frac{R}{c}) + \frac{r}{cR^2} \frac{\partial}{\partial t} i(z', \tau = \frac{R}{c}) \right] \]

\[ R = \sqrt{r^2 + (z-z')^2} \]

![Fig. 1. Geometry used to calculate the EM field.](image)

B. Ground effect in the horizontal electric field

For distance not exceeding several kilometers, the perfect ground conducting assumption is a reasonable approximation for the vertical component of the electric field and for the azimuthal magnetic field. In fact, the contribution of the source dipole and of its image to these field components add...
constructively, and consequently, small variations in the image field due to the finite ground conductivity will have little effect on the total field. However, the horizontal (radial) component of the electric field radiated by lightning is appreciably affected by the finite ground conductivity.

**Cooray-Rubinstein formula:**

One of approximation proposed in the literature, which appears to be one of the simplest and most accurate, is the Cooray-Rubinstein formula \([2], [3], [4]\):

\[
E_f(r, z, jw) = E_{r\phi}(r, z, jw) - \frac{\sqrt{\mu_0}}{\sqrt{\varepsilon + \sigma_z / j\omega}}
\]

Where \(E_{r\phi}(r, z = 0, jw)\) and \(H_{r\phi}(r, z = 0, jw)\) are the Fourier transforms of the horizontal component of the electric field at height \(z\), and the azimuthal component of the magnetic field at ground level respectively, both calculated assuming a perfect conducting ground.

The horizontal electric field has been computed using the Cooray-Rubinstein formula is presented in Fig 2, where it is calculated at different distances 100, 500, 1500 m, assuming \(\sigma\) (ground conductivity)=10\(^{-2}\) S/m, and \(\varepsilon\) (relative permittivity of the ground)=10. The obtained results are conformed to those published in [5].

The Cooray-Rubinstein formula permits to obtain satisfactory approximation to the horizontal electric field for some significant cases: in particular, it reproduces the positive, bipolar and negative polarities.

### III. FIELD-TO-TRANSMISSION LINE COUPLING MODEL OF INDUCED OVERVOLTAGE

For solving the problem of lightning EM field coupling to an overhead line, we use the antenna theory, the quasi-static approximation, and the transmission line theory (TL).

In this paper, the last one will be used due to the fact that it is the most appropriate for the problem of interest, most of the used coupling models being based on it. Coupling models can be expressed in different but equivalent formulations, the difference between these formulations (coupling models) lies essentially in the representation of the source terms as a function of exciting EM field components \([2], [4]\), which is given by the sum of the field radiated by the channel dipoles and the ground reflected field, both terminated in absence of the overhead line. The total field is given by the exciting field and the scattered field that is the field produced by the line response.

In this paper, we will use the Agrawal model which is most popular in EMC and power literature.

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Fig. 2. Horizontal electric field calculated at 6 m above a finite ground conductor \(\sigma=10^{-5}\) S/m, and at distance respectively, a) 100 m, b) 1000 m, c) 1500 m.

### A. Infinite ground conductivity

Let's consider first the simple case of lossless wire with radius \(a\) placed at height \(h\) above a perfectly conducting ground and its terminations \(Z_A\) and \(Z_B\) see Fig.3.
The line is illuminated by an external EM field.

\[ \frac{\partial u^x(x,t)}{\partial x} + Ri(x,t) + L' \frac{\partial^2 i(x,t)}{\partial t^2} = E_s(x,h,t) \] (6)

\[ \frac{\partial i(x,t)}{\partial x} + Gu^x(x,t) + C' \frac{\partial u^x(x,t)}{\partial t} = 0 \] (7)

\[ u^x(x,t) = u(x,t) - u^e(x,t) \] (8)

Where \( u^e(x,t) = \int_0^h E^e_z(x,h,t)dz \approx -hE^e_z(x,0,t) \) is the exciting voltage and \( E^e_z(x,h,t) \) and \( E^e_z(x,h,t) \) are the horizontal (along the conductor) and vertical components of the exciting electric field, respectively.

The terminal conditions in terms of the scattered voltage and the total current are given by:

\[ U^x(0) = -Z_A I(0) + \int_0^h E^e_z(0,z)dz \] (9)

\[ U^x(L) = Z_B I(L) + \int_0^h E^e_z(L,z)dz \] (10)

B. Lossy ground

The Agrawal coupling equation for the case of line above an imperfectly conducting ground can be written as:

\[ \frac{\partial u^x(x,t)}{\partial x} + Ri(x,t) + L' \frac{\partial^2 i(x,t)}{\partial t^2} + \xi \frac{\partial i(x,t)}{\partial t} = E_s(x,h,t) \] (11)

\[ \frac{\partial i(x,t)}{\partial x} + Gu^x(x,t) + C' \frac{\partial u^x(x,t)}{\partial t} = 0 \] (12)

Where \( \otimes \) denotes convolution product and \( \xi \) is called the transient ground resistance defined as:

\[ \xi(t) = F^{-1} \left[ \frac{z(t)}{j\omega} \right] \]

\[ = \min \left\{ \frac{1}{2\delta h} \frac{\mu_0}{\varepsilon_0^{\text{eff}}} \frac{\mu_0}{\pi \tau_g} \left[ \frac{\tau_g}{t} + \frac{1}{4} \exp \left( \frac{\tau_g}{t} \right) \text{erfc} \left( \frac{\tau_g}{2\sqrt{t}} \right) \right] \right\} \]

In which, \( \tau_g = h^2 \mu_0 \sigma_g \) and \( \text{erfc} \) is the complementary error function \( \text{erfc}(z) = \frac{2}{\sqrt{\pi}} \int_z^\infty e^{-u^2} du \).

IV. Validation of the Proposed Model

We have developed a finite difference time-domain computer code to calculate voltages and currents induced by lightning return stroke on overhead line above an imperfect conducting ground.

Fig. 4 shows the induced voltage at the end of a 1 km long overhead line terminated by its characteristic impedances and excited by a side stroke 50 m from the line center. The lightning current is calculated using the MTL model, assuming a channel-base current typical of subsequent return strokes (12 kA peak amplitude, 40 kA/µs maximum time-derivative), a return stroke velocity of 1,3.108 m/s, and a decay constant \( \lambda \) of 1.7 km. We have also presented, in the same figure, the lightning induced overvoltage obtained for the same configuration but considering the ground as an imperfect conductor with \( \sigma = 0.001 \text{ S/m} \).

Our simulations give results similar to those published in [7], [8], [9] where it can be seen that the ground conductivity significantly affects the induced voltage in magnitude, shape, and polarity.

\[ \sigma = \text{infinite} \]

\[ \sigma = 0.001 \text{ S/m, } \varepsilon = 10 \]

\[ \text{Voltage [kV]} \]

\[ \text{Time [µs]} \]

Fig. 4. Lightning induced voltage at the extremity

In Figure 5, we can see that the ground resistivity can increase or decrease the magnitude of the induced voltages depending on the stroke location and the observation point along the line.
V. RESULTS AND DISCUSSION

In our calculation examples, the two stroke locations, as shown in Figs. 6, will be used. The line terminations are matched i.e., terminated by the characteristic impedance.

Fig. 7(a and b) shows respectively a simulation of the induced voltage for an end and aside stroke at distance 100 m from a 1000 m long and 10 m high overhead line. The simulation taking a lossy ground into account is compared with a lossless simulation. The lightning current parameters are 1.9 $10^5$ m/s and $\lambda=1700$ m and $H=7500$ m. From Fig. 7, we observe that a line terminated by its characteristic impedance and excited by an end stroke is very sensitive to lossy ground effects.

Figs. 7 show that the end stroke configuration is very sensitive to loss effects while the side stroke configuration is less sensitive. The reason for this is that the latter configuration is less dependent on the horizontal field contribution and the loss effect on the total voltage will actually decrease with increasing distance until the loss effect on the incident voltage becomes important.

VI. CONCLUSION

Lightning induced-voltages on lines are computed using MTL model for the return stroke field calculation and the Agrawal et al. coupling model for field-to-transmission line coupling calculations.

Calculated voltages induced by a typical subsequent return stroke presented in Section V have shown that for lines whose length does not exceed a certain ‘critical’ value (typically 2 km), the surge propagation along the line is not appreciably affected by the ground finite conductivity which, therefore, can be neglected in the computation process.

REFERENCES


