Simplified Partial State Fuzzy-PID Control on Nonlinear Wheel Balancing Robot

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Abstract—This paper deals with the Fuzzy-PID control on Nonlinear Wheel Balancing Robot. A simplified T-S Fuzzy-PID scheme that will be based on the signed distance method has been proposed. Simplification process of ‘conventional’ Fuzzy-PID to ‘simplified’ Fuzzy-PID scheme through 3D vector projection was demonstrated. Without reducing the performance of conventional Fuzzy-PID control, the number of fuzzy rules was greatly reduced, which eventually minimized the computational burden of the processor. Besides, the proposed controller ensures the control energy was within permissible limit and it also eliminates the effects of nonlinearity when the system was only controlled by PID controllers. Finally, the performance of proposed controller is compared to the performance of conservative Fuzzy-PID and PID schemes via simulation.

Index Term—Nonlinear Wheel Balancing Robot, Simplified Fuzzy-PID scheme, Signed distance method.

I. INTRODUCTION

Nonlinear Wheel Balancing Robot (NWBR) is an innate unstable system. Therefore, this robot has to be continuously actuated by its motors to balance its main body from toppling. Thus, various control schemes such as linear [2-4], nonlinear [5-7], adaptive [8-10] and hybrid [11-13] controllers had been proposed to stabilize the robot. Linear control schemes had a low computational burden; however, these control signals are often much interference from disturbance and measurement noise. Moreover, its control performance degrades due to the effects of nonlinearity when NWBR is operated in larger tilt angles. In contrast, nonlinear and robust schemes are more robust to the aforementioned conditions. Thus these schemes require high computational effort from the system and they are extremely time consuming [14, 15].

As a nonlinear scheme, intelligent control scheme that is based on Fuzzy logic control (FLC) encounters similar issue [16]. Initially, one input-one output FLC controller was realized to stabilize the NWBR system [17]. Each of the input and output has five and seven fuzzy sets. There are some study that have considered the error and the rate of error of tilt angle as the inputs of FLC [18], and others [19, 20] that have designed this PD-Fuzzy controller as a balancing controller. Furthermore, when controlling two or more states of the NWBR system, designing sub-FLC can evade the implication of a complex FLC scheme [21]. This method diminished the required rules from 625 fuzzy rules in single FLC to 25 fuzzy rules in each of two FLCs. Further, the performance of PD-Fuzzy controller was compared to the performance of the PD controller when the centre of gravity (COG) of main body varied due to the effect of load disturbance [22]; FLC yielded faster stabilization. Alternatively, an implementation of PD-fuzzy controller and PD controller (respectively playing the role of position and balancing controllers) can reduce the computational load [23]. Meanwhile, a higher number of rules are needed to improve the control performance of the Fuzzy scheme [24], and consequently, 7 by 7 fuzzy set of rules were considered for each position FLC and balancing FLC [25]. By using incomplete rule approach, the number of rules was decreased to 35 fuzzy rules. Unfortunately, the controlled NWBR produces unexpected fluctuation and overshoot because the implemented FLCs had been realized via PI Mamdani law.

T-S Fuzzy scheme is also deployed to control NWBR. Whenever T-S fuzzy model of NWBR was available, Parallel Distributed Compensation (PDC) technique [26] could be adapted [27]. This work employed quadratic stabilization law and applied fuzzy observer to estimate the unavailable direct measure states. The inclusion of observer increased the dynamic of the system and thus producing more oscillatory state responses. In addition, the input of balancing PDC FLC can be gained by state feedback [28]. In other research, the gains were also tuned by considering LMI constrains [29]. The study conducted a comparative assessment between the controlled NWBR by LQ with linear observer, PDC FLC1 that comprising 16 T-S fuzzy models with fuzzy observer and PDC FLC2 that comprising 4T-S fuzzy model with fuzzy observer. Both PDC FLCs were more robust than the linear controller because the linear controller was unable to handle large uncertainties and the stability was lost as well. Yet, this mentioned work demonstrates PDC FLC as the only balancing controller. Again, when designing three FLCs to fully control the NWBR system, the PDC FLC was only applicable in fuzzy balanced standing control (FBCS) [30]. The other two FLCs-fuzzy travelling and position (FTPC), and fuzzy yaw steering

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control (FYSC)-were realized by using 7 by 7 fuzzy PI Mamdani rules. If PDC FTPC was also realized, this second term of control disappears and only FBSC is left in operation when the main body of NWBR in vertical position. Eventually, the robot will always remain at the starting point. Thus, to realize full control of the NWBR system by using T-S Fuzzy scheme, this paper proposes simplified Fuzzy-PID scheme that is called Simplified Partial state Fuzzy-PID controller (SPS-FPID). At first, this work designs Partial state Fuzzy-PID controller (PS-FPID). Unlike conventional Fuzzy control, the proposed PS-FPID scheme takes the error, summation of error and rate of error as the inputs; hence, improves the fuzzy control performance. Furthermore, using signed distance method [31], the applied PS-FPID controller can be simplified to SPS-FPID which tremendously decreases the computational burden of the processor. Lastly, via simulation, results show the proposed controller outperforms the others equivalent controller.

II. DYNAMIC MODEL OF NWBR

A NWBR consisted of an intermediate body, B, and two wheels, W_L and W_R. Other important points to note are the COG of the main body, G, the centre of right wheel, W_R, and the centre of left wheel, W_L where the gravitational force acts on them. Another vital point is c_b where force, F_b, is exerted on the body by two wheels. Fig. 1 shows a free body diagram of the NWBR travelling from point b to c. The NWBR can be modelled by using the Newtonian approach [32], its parameters are tabulated in Table 1. Firstly,  \( \dot{x}_{bc} \) is defined as a linear velocity and consider following condition:

\[
\begin{align*}
\dot{\phi}(t) &= \dot{\phi}(t), \\
\ddot{v}_w^c &= \dot{x}_{bc} \hat{T} \\
\ddot{\alpha}_c^G &= \ddot{\phi}_{G} \text{ and} \\
\ddot{\alpha}_b^G &= \ddot{\phi}_b.
\end{align*}
\]

The force and torque that are translated by the wheels to the main body at point \( c_b \) are

\[
\begin{align*}
\ddot{F}_b &= \dot{F}_{wr} + \dot{F}_{wl} = \frac{\tau_{wr} + \tau_{wl}}{R} \hat{T} \text{ and} \\
\ddot{x}_b &= (\tau_{wr} + \tau_{wl}) \frac{L}{R} \hat{j}.
\end{align*}
\]

The \( \dot{F}_{wr}, \dot{F}_{wl}, \tau_{wr} \) and \( \tau_{wl} \) are the generated forces and torques by motors on the right and left wheels. The acceleration of the main body with respect to point \( O \) can be formulated as

\[
\begin{align*}
\ddot{a}_c^G &= \frac{d}{dt} \ddot{v}_w^G + \frac{d}{dt} \left( \ddot{\alpha}_c^G \times \dot{v}_w^G \right) + \frac{d}{dt} \ddot{v}_c^G \\
&= \ddot{a}_c^G + \ddot{\alpha}_c^G \times \dot{v}_c^G + \ddot{v}_c^G + \ddot{\alpha}_c^G \times \dot{v}_c^G + \ddot{\alpha}_c^G \times \ddot{v}_w^G + \ddot{v}_c^G \\
&= \left( \ddot{x}_{bc} + d \cos \phi - d \sin \phi \right) \ddot{v}_c + \left( d \sin \phi + d \cos \phi \right) \ddot{v}_c^G.
\end{align*}
\]

where \( \ddot{v}_w^G = \ddot{v}_r^G + \ddot{v}_y^G, \ddot{a}_c^G = \frac{d}{dt} \ddot{v}_c^G \) and \( \ddot{v}_c^G \) is in vector form. The \( \ddot{v}_r^G \) and \( \ddot{v}_y^G \) are the velocity \( \ddot{v}_r \) in \( x \) and \( y \) direction respectively. The exerted force by the wheels can be solved from (4),

\[
m_b \ddot{a}_b^G = \ddot{F}_H + \ddot{F}_V + m_b \ddot{g}.
\]

The \( \ddot{F}_H \) and \( \ddot{F}_V \) are experienced horizontal and vertical forces on the wheel by the main body. Based on Newton second law, the following force and torque equations are obtained:

\[
\begin{align*}
\ddot{F}_H &= \frac{m_w}{L} \ddot{I}_{wl} + \ddot{F}_b - \ddot{F}_H \text{ and} \\
I_{b3} \ddot{\alpha}_3 + I_{b2} \ddot{\alpha}_2 &= \ddot{\tau}_g + \ddot{\tau}_b - \ddot{\tau}_{mb},
\end{align*}
\]

where \( I_{b3}, \alpha_3, \alpha_2, \tau_g \) and \( \tau_{mb} \) are inertia of each wheel in \( n_3 \) directions, tilting angular acceleration, steering angular acceleration, torque of main body at COG due to the gravity and yaw torque of the whole system. The following three equations of motion were obtained by appropriately solving (5) in \( \ddot{f}, \ddot{j} \) and \( \ddot{k} \) vector.

\[
\begin{align*}
\ddot{F}_b &= \ddot{F}_{wr} + \ddot{F}_{wl} = \frac{\tau_{wr} + \tau_{wl}}{R} \hat{T} \text{ and} \\
\ddot{x}_b &= (\tau_{wr} + \tau_{wl}) \frac{L}{R} \hat{j}.
\end{align*}
\]

Table 1:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_b )</td>
<td>mass of body</td>
<td>15kg</td>
</tr>
<tr>
<td>( m_w )</td>
<td>mass of wheel</td>
<td>0.42kg</td>
</tr>
<tr>
<td>( L )</td>
<td>half of lateral body</td>
<td>0.2m</td>
</tr>
<tr>
<td>( R )</td>
<td>radius of wheel</td>
<td>0.106m</td>
</tr>
<tr>
<td>( I_{b2} )</td>
<td>inertia of main body in ( n_2 ) directional</td>
<td>0.63kgm²</td>
</tr>
<tr>
<td>( I_{b3} )</td>
<td>inertia of main body in ( n_3 ) directional</td>
<td>1.12kgm²</td>
</tr>
<tr>
<td>( g )</td>
<td>Gravity force</td>
<td>9.81ms²</td>
</tr>
<tr>
<td>( d )</td>
<td>distance between point C and COG of main body</td>
<td>0.212m</td>
</tr>
</tbody>
</table>
\[ (m_b + 3m_w)\ddot{x}_{bc} + m_b d \cos \theta \ddot{\theta} = \frac{\tau_{wr} + \tau_{wl}}{R} + m_b d \sin \theta (\dot{\phi}^2 + \dot{\theta}^2). \]

\[ m_u d \cos \theta \ddot{x}_{bc} + (m_u d^2 + I_{bc}) \ddot{\theta} = m_b d \sin \theta + m_b d^2 \sin \theta \]

\[ m_u d^2 \sin \theta \cos \theta \ddot{\phi} + \frac{(m_u d \sin \theta)^2 + 2m_u L^2 + I_{bc} + 2I_{w2} + 2 \left( \frac{L \cos \phi}{R} \right)^2 I_{w3}}{\tau_{wr} + \tau_{wl}} \ddot{\phi} = \]

The inertia of wheels are considered as \( \omega = \frac{m_u R^2}{4} + m_u L^2 \) and \( I_{w3} = \frac{m_u R^2}{2} \). Solve (6) for \( \dot{x}, \dot{\theta} \) and \( \ddot{\phi} \). One may obtain state space representation of NWBR (7).

\[ \dot{x}(t) = A(x, t)x(t) + B(t)u(t) + f(x, t) \]

By linearizing NWBR and using parameters as in table 1, the following decoupled linear model of NWBR is obtained:

\[ \dot{x}(t) = A_i x(t) + B_i u_i(t) \]

The inputs of these subsystems are \( u_0 = \tau_{wr} + \tau_{wl} \) and \( u_\phi = \tau_{wr} - \tau_{wl} \).

III. SIMPLIFIED PARTIAL STATE FUZZY-PID

A. PS-FPID

To form a hybrid FLC and PID control, FLC is firstly designed to realize linear PID control behaviour. Then FLC with three inputs, namely error \( e \), change of error \( \dot{e} \) and integral of error \( \delta \), are considered. Using T-S fuzzy approach, a general rule for \( n \) inputs FLC can be realized as

\[ R_i : \text{IF} \ x_i \text{ is } F^i_{j1} \text{ AND } \cdots \text{ AND } x_n \text{ is } F^i_{jn}, \]

\[ \text{THEN } y_i = f_i(x) \]

where \( i = 1, 2, \ldots, m^n \) and \( m^n \) is the maximum number of \( IF-THEN \) rules of \( R_i \). \( m \) is the number of fuzzy linguistic variables of each fuzzy input variable. \( x_1, \ldots, x_n \) are the antecedents of input variables in its own universe of discourse \( U_{x_i}, \ldots, U_{x_n} \). \( F^i_{j1x_i} \) is a fuzzy set with a value of \( U^j_{x_i} \). The output function can be formulated as the summation of \( e \), \( \dot{e} \) and \( \delta \), as obtainable in a normal PID control.

\[ f_i(x) = f_i(e_i, \delta_i, \dot{e}_i) = e_i + \delta_i + \dot{e}_i \]

Then a rule table was established in three dimensional spaces and each fuzzy antecedent \( e \), \( \dot{e} \) and \( \delta \) has the following fuzzy set: \( NM, NS, Z, PS \) and \( PM \). If each of the antecedents is separated according to its neighborhood by an equal magnitude, thus the following fuzzy table as shown in Fig. 2 will be obtained. Further, to realize Fuzzy-PID control, the output function has to include PID parameters. Each of these PS-FPID controllers was illustrated as shown in Fig. 3 and its output function is

\[ f_i(x) = f_i(e_i, \delta_i, \dot{e}_i) = K_pe_i + K_i \delta_i + K_d \dot{e}_i \]

B. SPS-FPID controller

Three dimensions Fuzzy-PID table has 125 numbers of fuzzy rules as shown in Fig. 2. It can be indicated that the rules have multiple phase–planes in the same magnitude of the fuzzy output function in the diagonal direction as shown in Fig. 4. Every point on the diagonal surface \( S_i \) has a magnitude that is proportional to the perpendicular distance from the main diagonal surface \( S_Z \). This teopeítz structure allows FLC to be reformulated in such a way the output of the FLC is in single input function that called as “signed distance” \( d_s \). By the aid of Fig. 5, this Point-plane distance \( d \) can be solved by using projective matrix method. First, represent the main diagonal plane \( S_Z \) as
\[ S_Z : K_p e_z + K_i \delta_z + K_d \dot{e}_z + D = 0 \]  

Then solving the distance by projecting \( W \) to \( V \),

\[
d = |\text{proj}_V W| = \frac{|K_p \delta_z + K_i \dot{e}_z + K_d e_z|}{\sqrt{K_p^2 + K_i^2 + K_d^2}}
\]  

where \( V = [K_p, K_i, K_d]^T \) is a normal vector to surface \( S_Z \) and \( W = [e_z - e_p, \delta_z - \delta_p, \dot{e}_z - \dot{e}_p]^T \) is a vector that is pointing out from \( P_z \) to \( Q_p \). Dropping the absolute sign, this step gives the signed distance \( d_s \). Since the main diagonal surface is parallel to its neighboring surfaces, \( K_p \delta_z + K_i \dot{e}_z + K_d e_z = K_p e_z + K_i \delta_z + K_d \dot{e}_z \). 

Simplification can be made by considering point \( P_Z \) at the origin, where \( D = 0 \). Hence, ‘signed’ distance \( d_s \) can be obtained; where its value is positive if point \( Q_p \) is on the same side as the normal vector \( V \) and negative if it on the opposite side. The signed distance \( d_s \) is

\[
d_s = \lambda_{p1} e_z + \lambda_{p2} \delta_z + \lambda_{p3} \dot{e}_z
\]

where

\[
\lambda_{p1} = \frac{K_p}{\sqrt{K_p^2 + K_i^2 + K_d^2}} \\
\lambda_{p2} = \frac{K_i}{\sqrt{K_p^2 + K_i^2 + K_d^2}} \\
\lambda_{p3} = \frac{K_d}{\sqrt{K_p^2 + K_i^2 + K_d^2}}
\]

From the applied simplification method, one dimensional fuzzy table can be constructed as shown in Table 2. Thus each of the obtained SPS-FPID can be illustrated as in Fig. 6. The controlled system of NWBR with proposed SPS-FPID controller can be shown as in Fig. 7. Therefore, a total of three numbers of SPS-FPID controller are required for NWBR system where two controllers for balancing subsystem and one controller for heading subsystem. The proposed controller is used to control actual position \( x_a \), tilt angle \( \theta_a \) and heading.
angle $\phi_d$ of NWBR system. These controlled states need to track the desired position $x_d$, tilt angle $\theta_d$, and heading angle $\phi_d$ inputs trajectories. The control signals (torque) from FLC are

$$\tau_{wr} = 0.5(u_{q0} + u_{q2}) + 0.5u_{q}$$
$$\tau_{wr} = 0.5(u_{q0} + u_{q2}) - 0.5u_{q}$$

where $u_{q0} = f_{q0}(e_x, \delta_x, \dot{e}_x)$, $u_{q2} = f_{q2}(e_\delta, \delta_\delta, \dot{e}_\delta)$ and $u_q = f_q(e_\phi, \delta_\phi, \dot{e}_\phi)$.

### IV. Result and Discussion

#### A. Tuning the SPS-FPID

Firstly, the gains of proposed PS-FPID controller are tuned by the quadratic regulation scheme [33] where $Q$ and $R$ matrices are set to $Q_\theta = \text{diag}(83.112, 2.548, 5.229, 3.466, 0.001)$, $R_{\phi} = 1$, $Q_{\phi} = \text{diag}(10, 1000, 10)$ and $R_\phi = 1$. Thus following PID gains were obtained:

$$K_{PID1} = [-1.879 \ 0.579 \ 3.015]$$
$$K_{PID2} = [-44.088 \ 0.579 \ 9.022]$$
$$K_{PID3} = [12.331 \ 1.201 \ 3.287]$$

Further, the proposed PS-FPID has to be tuned to have similar control behaviour as PID controller. Based on the tuned gains of PS-FPID, proportional controller of $K_{PID3}$ has the highest magnitude of gain which results in the production of highest torque energy from this controller; hence, the fuzzy Position controller (PS-FPID$_1$) and Fuzzy steering controller (PS-FPID$_3$) is designed based on the design of balancing controller (PS-FPID$_2$). The PS-FPID$_2$ is designed to compensate the tilt angle error for up to $15^\circ$. Consequently, the initial universe of discourse of the inputs $e_x$, $\delta_x$, and $\dot{e}_x$ are from $-11.54$ to $+11.54$.

$$x_i = \frac{e_x \times 2\pi \times K_{PID3}}{360} = \frac{15 \times 2\pi \times 44.088}{360} = 11.54$$

However, to ease the distribution of the membership function of each fuzzy set in equal space, the universe of discourse is set to $\pm 13.2$. Further, these input membership functions were realized by triangular function where each of them intersects about half of its neighborhood and the output membership functions were realized by singleton function. The value of fuzzy linguistic variable $\{NM, NS, Z, PS, PM\}$ of the input is $\{-13.2, -6.6, 0, 6.6, 13.2\}$ and the fuzzy linguistic variable $\{3NM, 5NS, 2NM, 3NS, NM, NS, Z, PS, PM, 3PS, 2PM, 5PS, 3PM\}$ of the output is $\{-39.6, -33, -26.4, -19.8, -13.2, -6.6, 0, 6.6, 13.2, 19.8, 26.4, 33, 39.6\}$.

Using five linguistic variables for each input, each of fuzzy position, fuzzy balancing and fuzzy steering controllers has 125 fuzzy rules. This situation will cause a high computational burden to the system. Fortunately, utilizing sign distance method, the fuzzy rules were tremendously reduced to 13 fuzzy rules as shown in Table 2. Using the same type of membership functions, both input and output membership function of SPS-FPID controller can be shown as in Fig. 8.

Result shows that the obtained SPS-FPID is linear where the rate of control action is $'I'$'. To employ nonlinear fuzzy scheme and to improve the control performance, the fuzzy membership function is heuristically tuned by following preliminary consideration:

- If the error of tilt angle is less than $5^\circ$, the SPS-FPID has similar control action as linear PID scheme because the effect of nonlinearity is very small.
- If the error of tilt angle is within $5^\circ$ to $15^\circ$, the rate of control action of SPS-FPID is set to be larger than
linear control action.

- If the error of tilt angle is larger than $15^0$, the control action of SPS-FPID is saturated at certain value. This guarantees the SPS-FPID to produce permissible control energy (torque).

Finally, the tuned membership function of SPS-FPID is shown in Fig. 9. The values of fuzzy linguistic variable \( \{3NM, 5NS, 2NM, 3NS, NM, NS, Z, PS, PM, 3PS, 2PM, APS, 3PM\} \) of the input is \( \{-13.2, -11, -8.8, -6.6, -4.4, -2.2, 0, 2.2, 4.4, 6.6, 8.8, 11, 13.2\} \) and fuzzy linguistic variable \( \{3NM, 5NS, 2NM, 3NS, NM, NS, Z, PS, PM, 3PS, 2PM, APS, 3PM\} \) of the output is \( \{-13.2, -11, -8.8, -6.6, -4.4, -2.2, 0, 2.2, 4.4, 6.6, 8.8, 11, 13.2\} \). The fuzzy table of SPS-FPID is shown in Table 3. The motors used in this study can generate a maximum torque of 4 Nm; therefore the real value of \( u_{a1}, u_{a2} \) and \( u_6 \) in equation 16 is within ±8 Nm. Thus \( K_m \) was set to 0.6 to convert the output of FLC to the mentioned real value.

B. Computation time

This subsection shows a comparison of computation time between the NWBR that is controlled by PID (16), conventional Fuzzy-PD (CFPD) [22], PS-FPID and proposed SPS-FPID. The proportional and derivative gains of CFPD were also tuned by the quadratic regulation scheme. The testing of this NWBR with the respective controller was simulated ten times for 40s in each testing. The desired position and steering inputs were set as \( x_d(t) = (5\pi/3)/(1 + e^{0.3t-20}) \) and \( \phi_d(t) = -2\pi/3/(1 + e^{0.3t-20}) \). Table 4 shows the computation time and its standard deviation of the system of the mentioned controller. As expected, PID had the less computational time compare to fuzzy based controller as shown in Table 4. Conventional PS-FPID had the largest computational time, which is about fifteen times more than PID controller, but the computation time of CFPD and SPS-FPID was five and four times more than PID controller respectively.

C. Stabilization performance

In this section, stabilization behavior of NWBR was investigated when non-zero initial condition and disturbance occurred on the tilt angle state. The considered initial nonzero condition of tilt angle was \( 15^0 \) and the considered disturbance on tilt angle was \( 5^0 \) at time \( t = 50s \) for the duration of \( t_{psd} = 0.5s \). To show the effect of nonlinearity, the responses of linear WBR and NWBR with PID were simulated. Then, NWBR with CFPD, PS-FPID and SPS-FPID were simulated to examine their performance. Further, the tracking behaviors of NWBR that is controlled by CFPD, PS-FPID and SPS-FPID were also investigated. The NWBR is required to track the position input defined by

<table>
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<th>TABLE III</th>
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<tbody>
<tr>
<td>SPS-FPID TABLE</td>
</tr>
<tr>
<td>( d_i )</td>
</tr>
<tr>
<td>( u )</td>
</tr>
</tbody>
</table>

![Fig. 8. Input and output membership functions of SPS-FPID controllers.](image)

![Fig. 9. The tuned input and output membership function of SPS-FPID controllers.](image)
Fig. 10 shows the performance of NWBR deviated from the performance of linear WBR even it was controlled by the same PID controller. Compare to linear WBR, nonlinear NWBR took longer time to stabilize its position and tilt angle. The linear WBR generated rapid and higher torque response, but NWBR generated lower torque response. This is due to the nonlinearity factors that exist in the NWBR which degraded the performance of the system. The tuned liner PID controller has optimal performance only at equilibrium operation point; therefore, it cannot compensate this nonlinear effect when the robot is operated in larger tilt angle. As shown in Fig. 10, the performance of NWBR with SPS-FPID improved and was close to the behaviour of linear WBR. With SPS-FPID, the settling time of position and tilt angle of NWBR reduced, and the position state settled three times faster compare to the NWBR with PID controller. When large tilt angle causes the NWBR to move away from equilibrium operation point, the SPS-FPID initiates a nonlinear action to compensate; hence, it minimizes this nonlinear effect and further improves the performance of NWBR. The implementation of SPS-FPID also confirmed that the expended torque was less than $4\text{Nm}$ when stabilizing the main body.

Fig. 11 and 12 show the behaviour of NWBR in the existence of measurement noise in the tilt angle reading. The noise was assumed to be $\pm2.5^\circ$ in magnitude and frequency of $2\text{Hz}$. As shown in Fig. 11, the stabilization performance of all fuzzy based controllers was quite same, but the position response of CFPD was better. Meanwhile, Fig. 12 (a) shows that NWBR with the proposed SPS-FPID will have faster position tracking response and higher overshoot than NWBR with CFPD. Furthermore, Fig. 12 (b) shows that the NWBR with SPS-FPID will have less oscillation when stabilizing the robot as it moves to desired position. The result from both Fig. 11 and 12 showed the SPS-FPID maintained the performance of PS-FPID even when less number of fuzzy rules was implemented.

\[
x(t) = \frac{5\pi}{3} / (1 + e^{-0.3t - 2})
\]
V. CONCLUSION

A simplified Fuzzy-PID control scheme was proposed to control NWBR system for balancing and trajectory tracking application. Using singed distance method, proposed SPS-FPID had a small number of fuzzy rules and consequently reduced the computational load of the processing system. Results showed the proposed SPS-FPID successfully maintains the performance of a complex fuzzy control scheme, eliminates the effect on nonlinearity, and guarantees the control energy within the permissible range. Lastly, in future research, the designed controller can be programmed in the embedded NWBR to realize an autonomous NWBR system.

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