Real-Time Adaptive Predictive Control for Hybrid Thermoelectric Refrigerator System

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Abstract—This paper reports on control studies for Hybrid Thermoelectric Refrigerator (H-TER) system. The system is highly non-linear and exhibits varying model parameters and dead-time, hence the objective of the study is to investigate control strategies that are not based on detailed advance plant knowledge but allow for continuous adaptation of the controller to changing system dynamics. Using the generalized predictive controller (GPC), it is observed that the H-TER system is always stable even in the presence of randomly varying or step disturbance as well as dynamics model. The use of tuning parameters serve to improve system performance and to limit the controller from producing undesirable and unrealistically large, fast varying control signals. An interesting outcome from these studies is that controller design using GPC provides a more systematic approach is choosing the design parameters to achieve the desired performance specifications. However, it is also noted that the effects of non-linearity and varying time delay has not been investigated for this system. Under such situations, the design must involve the choice of prediction and control horizon, assumed to be fixed in this study. Then, it would mean more computation as the matrices involved would considerably increase in dimension.

Index Term—black box, adaptive, control, self-tuning, thermoelectric, refrigerator

I. INTRODUCTION

The essential feature of predictive controllers is the replacement of the problem of controlling the current system output by that of controlling a predicted value of the output at some future point. The objective being isolation of the effect of time delay, k, from the actual process dynamic to provide tighter control. It is well known in process control using conventional controllers that the presence of time delay or dead time which is often variable will cause retuning of conventional controllers to be done to prevent instability.

In this study considers a range of future predictions of output which lead to the generalized predictive controller (GPC) developed by [1]. It is expected that by extending the control horizon beyond the longest possible time delay will make the controller more robust against dynamics and work well for time varying systems. Besides, it has been proven to be capable of reducing the magnitude of control signal.

The plant that wants to control is Hybrid Thermoelectric Refrigerator (H-TER). H-TER is developed by configuration of two type thermoelectric device. Thermoelectric device is a kind of technology with solid state and no moving parts. The combination of cooling/heating ability makes them attractive for heating and cooling applications that use a small electrical energy. Thermoelectric couples are connected in parallel thermally and in series electrically, are integrated into modules. Typical thermoelectric modules (TEMs) contain from 3 to 127 thermocouples [2]. Thermoelectric devices are not the option for all cooling problem. Alternatively, it should considered in case the design requirement of the system include such as factor high consistency, small size or capacity, inexpensive, less height and intrinsic safety for risky electrical conditions [3-5]. These systems are energized by a DC power input. This H-TER system is very dynamic due to changing environment temperatures cause of the coefficient of the refrigerator is very low.

Numerous research studies are available on the modeling (steady state and distributed parameter) of thermoelectric refrigerator systems for design refrigerator purpose as, summarized by [6-10]. Due to the dynamics, time varying stochastic nature of H-TER system [11-13], the application of adaptive control techniques to H-TER system has an attention in this research.

II. HYBRID THERMOELECTRIC REFRIGERATOR SYSTEM

The main components of the proposed system, which were studied, consisted of an aluminium box (12 cm width x 12 cm length x 10 cm height with a thickness of 3.5 mm), two thermoelectric devices was applied: direct thermoelectric heat pump (operating 12 VDC, 2.6 Amp) and air to air thermoelectric heat pump (operating 12 VDC, 4 Amp), finned heat sink and fans (12 cm width x 12 cm length operating 12 VDC, 0.18A) as shown in Fig. 1.
The air-air thermoelectric heat pump device was cool the aluminium chamber by convection the cool air to the chamber. The direct thermoelectric heat pump was cool the chamber by transfer the heat from the aluminium chamber with touch to the thermoelectric device. In this study, 2 type of chamber was introduced there are aluminium (H-TER I) and stainless steel (H-TER II) to show the performance of the controller.

A digital-analog IO DAQ provide digital-analog signal channel allow for closed loop control through connection to MOSFET to control the input actuator (H-TER). The thermoelectric module operate directly from DC power, being controlled by a computer program to maintains the temperature inside the chamber by controlling the current, and automatically corrects for the temperature by means of a feedback loop. A resistance temperature sensor, called a RTD, was used to determine the temperature inside the refrigerator.

The energy balance in conservation equation for thermoelectric materials, there are three effect must be considered: the heat transferred by conduction (Fourier’s heat conduction law), the thermoelectric effect (Peltier effect), and the heat generated (Joule effect).

Conservative form:

$$\frac{\partial \rho CT}{\partial t} = \nabla \cdot (k \nabla T) - \nabla \cdot (\tau J T) + \xi J$$

Non-conservative form:

$$\frac{\partial \rho CT}{\partial t} = \nabla \cdot (k \nabla T) - \tau J \cdot \nabla T + \xi J^2$$

where: \(T\), temperature; \(J\), current density (electrical current per unit area); \(\tau\), Thompson coefficient; \(\xi\), electrical resistivity; \(C\), heat capacity; \(\rho\), density; \(k\), thermal conductivity.

Assuming uniform hot and cold side temperature the temperature distribute through it can be consider as one dimensional, and the current density through it as constant. Then, equation (2) is reduced to:

$$\frac{\partial \rho CT}{\partial t} = \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) - \tau \frac{1}{A} \frac{\partial T}{\partial x} + \xi \left( \frac{I}{A} \right)^2$$

where: \(I\), current through the TEM; \(A\), area of the TEM.

The solution of equation (3) gives the temperature distribution though the TEM, from its cold surface temperature to its hot surface temperature. The cold side temperature of the TEM has to be lower than the required cold temperature in order to pump heat from the cooling load, while the hot side temperature has to be higher than the ambient temperature in order to reject the heat pumped from the load plus the heat generated by the Joule effect.

III. GPC ALGORITHM FOR H-TER SYSTEM

The selection of an adequate thermoelectric device is a difficult process. The performance of thermoelectric devices relies on numerous variables including operating temperature, voltage and current supplied, performance of the heat sink on the hot side, and the power pumped on the cold side. Typically a thermoelectric system does not function at a specific circumstance. The changing temperature environment and various power degenerated make the thermoelectric system become dynamic. Therefore, a lumped model is adequate for the analysis of this complex thermoelectric system.

A. Development of GPC Algorithm

In GPC, the variance of the output can be minimized subject to a constraint on the variance when manipulated input increment. The controller is then designed to minimize the following criterion:

$$J(u,t) = E \left\{ \sum_{j=1}^{N} \left[ \Delta y_j(t) - r_j(t) \right]^2 + \lambda \sum_{j=1}^{N} \left[ \Delta u_j(t) \right]^2 \right\}$$

subject to: \(\Delta u(t+j) = 0\) for \(j > N_u\)

where

\(N_1\) = Minimum costing horizon
\(N_2\) = Maximum costing horizon
\(N_u\) = Control horizon
\(\Delta u(t+j)\) = increment of control input of the system
\(\lambda\) = weighting on control action increment

The minimization produces control signals \(u_1, u_2, \ldots, u_{N_1}; u_{N_1+1}, \ldots, u_{N_u}\); however, only \(u_1\) is actually applied. Since the new minimization problem is solved a time \(t+1\), this implementation is called Receding Horizon Control. Its use is expected to increase robustness of the controller in the presence of variable time delay as it treats the process as if the assumed delay is always true.

B. Process Model and Prediction

The model used should characterize the input/output dynamics and disturbances affecting the process. The auto-regressive moving average (ARMAX) model is given by:

$$A(z^{-1})y(t) = B(z^{-1})u(t-1) + C(z^{-1})\xi$$

where \(A(z^{-1}), B(z^{-1})\) and \(C(z^{-1})\) are as defined before.

This simple model structure is suitable for discrete computer application and can manipulated into a predictive form. The minimization of the cost function (4) is derived base on known values at time \(t\) and on the future values of the control increments by computing a set of \(j\)-step ahead predictions of the output \(y(t+j)\) which will minimize the cost function.
function. The Diophantine equation is used in deriving j-step ahead predictors and solving prediction theories. Consider the equation:

\[ C(z^{-1}) = E_j(z^{-1})A(z^{-1})\Delta + z^{-j}F_j(z^{-1}) \]  

(5)

where C and A are as defined previously, j the prediction interval and polynomial E and F are defined thus:

\[ E_j(z^{-1}) = 1 + e_{j1}z^{-1} + e_{j2}z^{-2} + \ldots + e_{jn}z^{-n} \]  

(6)

\[ F_j(z^{-1}) = f_{j1} + f_{j2}z^{-1} + \ldots + f_{jn}z^{-n} \]  

(7)

The order of \( E_j(z^{-1}) \) is j-1; so the noise components are all in the future and it is recognized as the unknown predicted error, while \( F_j(z^{-1}) \) is of degree max \( \{n_a, n_e\} \) and associated with the known part of the j-step ahead predicted disturbance.

From the process model of (5):

\[ y(t+j) = \frac{B(z^{-1})}{A(z^{-1})}u(t+j) + E_j(z^{-1})\xi(t+j) + \frac{1}{A(z^{-1})}\Delta \]  

(8)

Replacing \( \tilde{\xi}(t) \) from equation (5); (8) can be written as:

\[ y(t+j) = \frac{F_j}{C}y(t) + \frac{E_jB}{C}\Delta u(t+j) + E_j\tilde{\xi}(t+j) \]  

(9)

Lastly, define the vector consisting of future control increments \( \mathbf{u} \), with \( \Delta u(t+j) = 0 \) for \( j \geq n_a \), then:

\[ \mathbf{u} = [\Delta u(t), \Delta u(t+1), \ldots, \Delta u(t+N_a-1)]^T \]  

(10)

If the free response predictions over a range of j=1 to j=N is defined in a vector \( \mathbf{f}_r \), then:

\[ f_r = \left[ \overline{y}(t+1), \overline{y}(t+2), \ldots, \overline{y}(t+N_a) \right]^T \]  

(11)

From (11)

\[ y = \mathbf{Gu} + f_r \]  

(12)

where matrix \( \mathbf{G} \) consists of the impulse response parameters \( g_i \) of B/A. We can then write (6.2) in vector form:

\[ \mathbf{J} = \left( \tilde{y} - r \right) \left( \tilde{y} - r \right) + (\lambda \mathbf{u}^T \mathbf{u}) \]  

(13)

to give us the future incremental control vector \( \mathbf{u} \) as:

\[ \mathbf{u} = \left( \mathbf{G}^T \mathbf{G} + \lambda \mathbf{I} \right)^{-1} \mathbf{G}^T \left( \mathbf{r} - \mathbf{f}_r \right) \]  

(14)

where \( \mathbf{r} \) is a known reference signal:

\[ \mathbf{r} = \left[ r(t+1), r(t+2), \ldots, r(t+N_a) \right] \]  

(15)

IV. EXPERIMENTS AND RESULTS

A. Effect of Control Weighting

In Figure 2, 3 and 4 show the response of the H-TER I system with the model structure obtained from ARMAX. To study the effect of control weighting \( \lambda \) alone, the system is investigated under the step disturbances, load disturbance (40ml water) and considered with noise term model in 28°C-29°C dynamic ambient. Figure 2 shows good response with small variation in control signal, while for \( \lambda=0.01 \), the effect is observed in Figure 3 which shows greater variation in control signal but essentially no effect on the output response which display good response on the disturbance rejection characteristics. The control signal is large and oscillating, this is not an indication of good control. The Figure 3 shows better temperature response and the more interesting result is the significant reduction in the magnitude of control effort with the \( \lambda=0.1 \).
same magnitude of step disturbance are injected at \(k=0, 620,\) and \(2400\) in Figure 5, 6 and 7. The response when \(\lambda=0.01\) is shown in Figure 5, which still indicates capability of disturbance rejection with greater control effort as compared to results in Figure 6 and Figure 7. However, it is observed that the system reacts more strongly to each disturbance, considering the magnitude of disturbance is still same.

Figure 6 shows the response of the system when \(\lambda=1,\) subjected to the same load disturbance as before. Disturbance rejection is still good even though the step disturbance (at \(k=0, 620\) and \(2400\)) give rise to much large change in temperature response. Figure 7 show the temperature response displays less steady state error then the results in Figure 5 and Figure 6. The more interesting result is the significant reduction in the magnitude of control effort with this value of \(\lambda.\)

It is observed that in the presence of noise or uncertainties, the control effort increase considerably when weighting on control signal diminishes and this seldom desirable in practice as it would increase wear and tear on the mechanical part of the system. Therefore it is always recommended to assign.
control weighting of value $\lambda > 0$ on the incremental control signal.

In all cases considered thus far, temperature response to a load disturbance is rejected very well. This effect can be observed in Figure 8 which shows the variation of system parameter during the course of control. The parameter have not converged to the final value for particular period is occur when the real time system are always changes in the system dynamic and the priori knowledge in designing the adaptive controller give very good temperature response to the system.

### B. Effect of C Polynomial Term and Step Load Disturbance

The most control calculations are based on a linear model of especially lower order as has been done for in this study. Furthermore, the nature of an H-TER system investigated here is always subjected to disturbance, besides processing varying dynamic due to different operating characteristic at different times of the day. The effect of a polynomial C on the robustness of controller against two different dynamic model (H-TER I and H-TER II) and disturbance is investigated in this section.

This study chooses an estimated model of the same structure but different parameters as it was use from different H-TER (H-TER I and H-TER II) system using real time data, that the structure of the model is the same and it differ only in the parameter.

In order to study the effect of dynamics model, an estimated model which different chamber (H-TER I and H-TER II) is specified as another second order model with different dominant poles and zeroes. Then the different structures of C are applied to see the effect on control signal and system response. For the H-TER system modeled in this study, C is chosen to have following transfer function:

$$C = 1 + c_1 z^{-1} + ... + c_n z^{-n}$$

where $n = 1, 2, ...$ is the order of the polynomial.

The result in Figure 9 show the responds to step disturbances at $k=0, 560$, and $k= 1120$ on H-TER I model and the $C=1+C_1+C_2$ polynomial is use. It is obvious that the controller is capable of disturbance rejection even in step disturbance during the control. Even though the output response remains stable, the control signal is reduced but still relatively large and vigorous.

The use of first order C as is in this dynamic environment is capable of better disturbance rejection than $C=1+C_1+C_2$ parameter with small control effort. This is illustrated in Figure 10 where C=1+$C_1$ is used and step disturbance was injected at $k=0, 560$, and $k= 1120$. 
The performance of the GPC algorithm developed for H-TER I is tested with H-TER II. The first order \( C \) is used, the disturbance is rejected well even different H-TER system is implemented as observed in Figure 11. This time the control signal still reduced but relatively crucial compare to when the controller implemented on H-TER I. Step disturbance at three different instances are introduced and the system is seen to be enough to maintain good control over the entire control period.

CONCLUSIONS

It has been implemented that the H-TER system studied, the system with GPC controller is always show stability even in the presence of step disturbance. The use of tuning parameter serve to improve system performance and limit the controller from producing undesirable and unrealistically large, fast varying control signals.

In this chapter the performance of H-TER system incorporating a GPC controller in real-time condition has been studied in particular its ability to reject disturbances and its robustness against dynamics environment. Experiment results show that the controller is able of handling both problems with proper tuning its parameters. It is the opinion of the author that the controller design using GPC provides a more systematic approach in choosing the design parameters to achieve the desired performance specifications. However it is also noted that the effects of nonlinearity and varying time delay has not been investigated for this system. Under such situations, the design must involve choice of prediction and control horizons, assumed to be fixed in this study. Then, it would mean more computation as the matrices involved would considerably increase in dimension.

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REFERENCES


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