Modelling and Design Adaptive Control for Thermoelectric Refrigerator System

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Abstract —The objective of this study is to investigate the application of various techniques of adaptive control in industrial processes with the temperature maintained between 2°C to 8°C for the thermoelectric refrigerator (TER) system for vaccine chamber. The system is nonlinear and exhibits varying model parameters, hence the object of the study is to investigate control strategies that are not based on detailed advance plant knowledge but allow adaptation of the controller changing the system dynamics. The mathematical model has been identified for the TER system using input-output real-time system. The black box modelling approach is appropriately chosen since this is needed for the implementation of adaptive control. The TER system has been identified using Recursive Least Squares (RLS) method. A second order model is use to represent the system. The validation procedures show that the derived model is indeed a good enough representation of the TER system. Adaptive self-tuning control approaches of pole assignment with several variations have been considered.

Index Term— black box, adaptive, control, self-tuning, thermoelectric, refrigerator

I. INTRODUCTION
Thermoelectric device is a kind of technology with solid state and no moving parts. The combination of cooling/heating ability makes them attractive for heating and cooling applications that use a small electrical energy. Thermoelectric couples are connected in parallel thermally and in series electrically, are integrated into modules. Typical thermoelectric modules (TEMs) contain from 3 to 127 thermocouples [1]. Thermoelectric devices are not the option for all cooling problem. Alternatively, it should considered in case the design requirement of the system include such as factor high consistency, small size or capacity, inexpensive, less height, intrinsic safety for risky electrical conditions, and precise temperature control. These systems are energized by a DC power input.

In order to study the performance of a system, it is often necessary to establish its mathematical models. The choice of system identification method is based on understanding the method and the system characteristics. Numerous research studies are available on the identification of engineering systems as, summarized by [2-5].

Due to the nonlinear, time varying and stochastic nature of TER system, the application of adaptive control techniques to TER system has an attention in this research.

II. THERMOELECTRIC REFRIGERATOR SYSTEM
The main components of the proposed system, which were studied, consisted of a aluminium box (12 cm width x 12 cm length x 10 cm height with a thickness of 3.5 mm), two thermoelectric modules: direct air heat pump (operating 12 VDC, 2.6 Amp) and air to air heat pump (operating 12 VDC, 4 Amp), finned heat sink and fans (12 cm width x 12 cm length operating 12 VDC, 0.18A) as shown in Fig. 1.

A digital-analog IO DAQ provide digital-analog signal channel allow for closed loop control through connection to MOSFET to control the input actuator (TER). The thermoelectric module operate directly from DC power, being controlled by a computer program to maintains the...
temperature inside the chamber by controlling the current, and automatically corrects for the temperature by means of a feedback loop. A resistance temperature sensor, called a RTD, was used to determine the temperature inside the refrigerator.

The energy balance in conservation equation for thermoelectric materials, there are three effect must be considered: the heat transferred by conduction (Fourier’s heat conduction law), the thermoelectric effect (Peltier effect), and the heat generated (Joule effect).

Conservative form:
\[
\frac{\partial pCT}{\partial t} = \nabla \cdot (k \nabla T) - \nabla \cdot (J \tau T) + \xi J
\]

Non-conservative form:
\[
\frac{\partial pCT}{\partial t} = \nabla \cdot (k \nabla T) - \tau J \cdot \nabla T + \xi J^2
\]

where: \( T \), temperature; \( J \), current density (electrical current per unit area); \( \tau \), Thompson coefficient; \( \xi \), electrical resistivity; \( C \), heat capacity; \( \rho \), density; \( k \), thermal conductivity.

Assuming uniform hot and cold side temperature the temperature distribute through it can be consider as one dimensional, and the current density through it as constant. Then, equation (2) is reduced to:
\[
\frac{\partial pCT}{\partial t} = \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) - \tau \frac{\partial T}{\partial x} + \xi \left( \frac{I}{A} \right)^2
\]

where: \( I \), current through the TEM; \( A \), area of the TEM.

The solution of equation (3) gives the temperature distribution though the TEM, from its cold surface temperature to its hot surface temperature. The cold side temperature of the TEM has to be lower than the required cold temperature in order to pump heat from the cooling load, while the hot side temperature has to be higher than the ambient temperature in order to reject the heat pumped from the load plus the heat generated by the Joule effect.

## III. MODELING

The selection of an adequate thermoelectric device is a difficult process. The performance of thermoelectric devices relies on numerous variables including operating temperature, voltage and current supplied, performance of the heat sink on the hot side, and the power pumped on the cold side. Typically, a thermoelectric system does not function at a specific circumstance. The changing temperature environment and various power degenerated make the thermoelectric system become dynamic. Therefore, a lumped model is adequate for the analysis of this complex thermoelectric system.

### A. Recursive least squares method

Parameter estimation is the determination of values of parameters that govern the dynamic behaviour of a process with the structure of the process model assumed to be known. A recursive algorithm has the advantage of providing significant savings in computation as non-recursive version would need the recalculate the whole estimate, requiring the storage of all pervious data.

Fig. 2 provides a useful way to visualize the estimation process where new input/output data become available at each sample interval. The model based on past information and stored in matrix \( \theta(k-1) \) is used to obtain and estimate of current output \( y(k) \). This will then compared with the observed output \( y(k) \) to generate an error \( e(k) \). Subsequently, this used to generate an update to the model which makes a correction to \( \theta(k-1) \) to approach the new \( \theta(k) \).

![Recursive estimation mechanism](image)

### B. Autoregressive with exogenous input model (ARX model)

The input-output relationship of an ARX model is given by a linear differential equation as follows:
\[
y(t) + a_1 y(t-1) + \ldots + a_{na} y(t-na) = b_0 u(t-d) + \ldots + b_{nb} u(t-d-nb) + e(t)
\]

(4)

It is also known as an equation error model. Since the noise \( e(t) \) enters as a direct error in equation (4). The adjustable parameters of this structure are expressed as below:
\[
\theta = [a_1, a_2, \ldots a_{na}, b_0, b_1, \ldots b_{nb}]
\]

Given that
\[
A(q^{-1}) = 1 + a_1 q^{-1} + \ldots + a_{na} q^{-na}
\]

and
\[
B(q^{-1}) = b_0 + b_1 q^{-1} + \ldots + b_{nb} q^{-nb}
\]

(7)

where \( q^{-1} \) is the backward shift operator, the following input-output model structure is informed.
\[
A(q^{-1}) y(t) = B(q^{-1}) u(t) + e(t)
\]

(8)

\[
y(t) = \frac{B(q^{-1})}{A(q^{-1})} u(t) + \frac{1}{A(q^{-1})} e(t)
\]

(9)

## IV. CONTROLLER DESIGN

The control objective for an TER system requires that the output \( y(t) \) to follow a reference value in some predetermined way after the occurrence of a significant environment temperature changing (disturbance) and to reject random disturbance which corrupt the output.

### A. Pole Assignment controller

An approach in self-tuning controller is to use pole assignment as the control objective. This method is known
to work well even with non-minimum phase systems. The problem is then to design a controller which regulates the output by automatically moving the closed loop poles of the system from their open loop position to the desired location specified by polynomial \( T(z^t) \) given by:
\[
T = 1 + t_1z^{-1} + t_2z^{-2} \ldots + t_nz^{-n}
\]  
(10)

Therefore, a second order response is required and the desired poles will consist of the two zeroes of the second order polynomial of \( T(z^t) \):
\[
T = 1 + t_1z^{-1} + t_2z^{-2}
\]  
(11)

The values \( t_1 \) and \( t_2 \) should be chosen to achieve the desired transient performances which are specified in terms of the system’s damping characteristics and natural frequency. Using transient response technique, these can be written as:
\[
t_1 = -2\exp(-\xi\omega_s\tau_s)\cos(\omega_s(1 - \xi^2)^{1/2})
\]  
(12)
\[
t_2 = \exp(-2\xi\omega_s\tau_s)
\]  
(13)

where \( \xi \) is the damping ratio and \( \omega_s \) is natural frequency of the desired closed loop second order transient response. It is often best to specify \( \xi \) and \( \omega_s \) such that the system achieves desired properties. The damping factor is normally chosen in the interval 0.5 to 0.8 while the natural frequency is chosen based on the demands on the rise time and the settling time.

The TER system is described by the equation derived in the previous ARX model i.e. equation (4). The controller polynomial \( F, G, H \) are given by:
\[
F = 1 + f_1z^{-1} + \ldots + f_nz^{-n}
\]  
(13)
\[
G = g_1z^{-1} + \ldots + g_nz^{-n}
\]  
(14)
\[
H = h_1z^{-1} + \ldots + h_nz^{-n}
\]  
(15)

Combining the controller and the system equation yields the closed description
\[
(AF + z^{-1}BG)y(t) = z^{-1}B\Delta r(t - 1)
\]  
(16)

The system performance is defined by its closed loop characteristic polynomial which is given by \((AF + z^{-1}BG)\). In order to achieve pole assignment objective, the closed loop poles are then assigned to their desired locations, specified by \( T \), by selecting controller parameters \( F \) and \( G \) according to the polynomial identity:
\[
AF + z^{-1}BG = T
\]  
(17)

The solution to this equation set is unique if \( A \) and \( B \) are and the degrees of \( F \) and \( G \) are selected to satisfy the following reliability conditions:
\[
n_f = n_g
\]  
(18)
\[
n_f = n_g - 1
\]  
(19)

### B. Incremental Pole Assignment controller

Industrial process control schemes frequently specify an incremental control action whereby the control algorithm will output a signal sequence
\[
\Delta u(t) = u(t) - u(t - 1)
\]  
(22)

while \( u(t) \) is actually applied to the system.

This can be express in the alternative incremental form
\[
\bar{\Delta}y(t) = B\Delta u(t - 1)
\]  
(23)

where
\[
\bar{\Delta} = (1 - z^{-1})A = \Delta A
\]  
(24)

Now assume a controller of the form
\[
\Delta u(t) = -\frac{G}{F}(y(t) - r(t))
\]  
(25)

with \( G, F \) selected to satisfy the pole assignment identity
\[
FA = z^{-1}BG = T
\]  
(26)

### V. EXPERIMENTS AND RESULTS

#### A. Step response TER system

A simple method to characterize the dynamic properties of the estimated model is by examining the transient response of the system. This is step response in the time domain. The open-loop temperature response curve is the dynamic response of the TER system toward the step input signal of 10% PWM cycle; correspond to a increase of about 5°C, from the current temperature.
This is shown in Fig. 3 where the response of the real time TER model has been presented. It can be seen that the model provide good transient with without overshoot and slow settling time.

For the temperature considered, the computer program has been set up so that initially, the TER system is run at a steady state temperature value before a step input signal is applied to the TER at time $t=0.6$ sec of the data acquisition period. The curve of figure exhibits a first-order system response. Furthermore, looking at each of the step response curve of the TER, it can be seen that there is a delay between the input signal and the TER temperature response. This is due to the transport delay and the combination of the system. This is time delay is approximated from the step response curve and considered as the system delay of the TER. It will be taken into the account in the designing of the PRBS signal. Table show the approximate values of the TER system time constant, settling time, and the system delay for the respective temperature range.

### B. Real time identification of TER system

In a real-time, online environment of parameter estimation, the magnitude and sequence length of the PRBS signal should be selected properly as too small a magnitude of the input signal will not be simulating sufficient and the system response will be improperly represented which will consequently cause parameter estimates is to be inaccurate. The choice of input signal has a very substantial influence on the observed data [7]. On the other hand, too large a value will drive the system into non-linearity and even instability. The signal is varied between two levels and it can switch from one level to the other at time $t=0, \Delta t, 2\Delta t, 3\Delta t, \ldots$ where $\Delta t$ is the time interval or clock period. In this experiment, the clock period is calculated according to equation (27):

$$\Delta t = (0.2 \leq 0.5) \times T_c$$  \hspace{1cm} (27)

However, since there is a delay in the system response, this must taken into account in the calculation of the clock period must consider it. The clock period chosen in designing the PRBS is then become:

$$\Delta t = (0.2 \leq 0.5) \times T_c + \text{time delay}$$  \hspace{1cm} (28)

Referring to step response of TER system, using equation (28), the clock period $\Delta t$ for the temperature range is calculated as follows:

$$\Delta t = (0.2 \leq 0.5) \times 750 + 18.1 = (168.1 \leq 393.1) s$$  \hspace{1cm} (29)

From this, the clock period chosen for temperature range in this study is $168.1 s$.

The determination of sequence length of PRBS input should consider the total duration time of the signal to exhibit a complete transient behavior of the system. Choosing the settling time of the TER system to be approximately equal to the sequence period $T$ of the PRBS signal, the minimum PRBS sequence length can be determined by:

$$T = \Delta t \times N$$  \hspace{1cm} (30)

Using settling time $2800$ s from the step response,

$$N = \frac{2800}{168.1}$$  \hspace{1cm} (31)

However, the sequence length from this calculation is too short. By using trial and error procedure, a PRBS signal with maximum length sequence of $31$ with time period of $1s$ used during data collection. The input magnitude of the PRBS signal is chosen to be between 80 and 90 is chosen for the PRBS signal. The procedure is to initially let the engine run at steady state value according to the temperature range under study. After a prescribed time elapsed, PRBS signal is injected into the system that varies the system output within 80 and 90 from the steady state temperature.
Fig. 5 shows the system parameter estimates using the RLS algorithm with a constant forgetting factor 0.98. The noise has been assumed to be white. It can be seen that convergence is fast and more significantly, the parameter value are not biased. The use of RLS estimation is in unbiased estimates of system. This can be seen from Fig. 5 (parameter system). The parameters vector given below shows only slight variation from the expected values of the parameters.

C. Validation of TER system

The use of residual analysis is another aid for validation purposes. Residual analysis consists two parts:

- Testing cross-correlation function between input and the residuals, that is the difference between the real output data and the output reproduced by the model. If the model obtained is good, the residual should be independent of the input and the cross-correlation function should lie within the specified confidence interval.

- Auto-correlation test for the residual to ensure that the residuals are mutually independent. Again, for an ideal model this function should be entirely inside the confidence interval.

D. Control system performance of TER system

Fig. 7 shows the response of TER system using a first order pole assignment controller (PAC), with the value for $t_1$ equal 0.199. Temperature output is 4°C in approximately 3000s with the steady state error 1°C. With a second order controller, the performance much better at the second instant as shown in Fig 8.

This shows that while a second order controller display better adaptability, the settling time for the initial transient is much smaller due to more computational of controller parameters. However, it indicates that a first order or a simplifier controller is capable of producing comparable result without more elaborate computation.

CONCLUSIONS

Identification of thermoelectric refrigerator control system has been carried out successfully. An input-output model is appropriately chosen since this suitable for the implementation of adaptive controllers. RLS estimation technique is thus found suitable to estimate the system parameters. A second order model is found to adequately represent the system as it gives best fit with better properties. It is expected that the
performance of the controller will not be significantly degraded by the model order reduction while the advantage is that the controller design will be greatly simplified. Furthermore, validation procedures show that the derived model is indeed a good enough representation of the TER system.

This study has developed and implemented various adaptive self-tuning PAC techniques for a TER system with several variations introduced. All the algorithms display varying degrees of ability in maintaining required temperature in dynamic environment. The second order PAC produces the best performance in terms of overshoot and settling time. Since modeling of the TER system reveals that the system parameters are indeed time varying during the course of its ambient conditions, this issue needs to be specifically addressed and will be further investigated using the predictive controller.

For further study of this system, a stochastic model can be introduced to get the comparison to deterministic model of this system. A stochastic model based controller also can be applied to this TER system to find the best controller for the system.

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REFERENCES


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