Finite Element Modeling of Brick-Mortar Interface Stresses

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Abstract
The splitting failure of brickwork under compressive loads has in general been associated with the transverse strain of the mortar joint due to the difference in the elastic properties of the brick and mortar. While the load causing failure is compressive, the stresses initiating it would appear to be tensile. The magnitude of such tensile stresses in this research was seen to depend on the elastic properties such as modulus of elasticity (E) and poisson’s ratio (ν) of the two materials concerned. Thus by carrying out an elastic analysis of the brick-mortar couplet and taking into account the different elastic properties of the brick and mortar elements, a relationship between the interface tensile stress and elastic properties was obtained. This relationship shows the controlling influence of brick-mortar elastic properties on the splitting failure of brickwork.

Keywords: Finite element method, masonry, brick-mortar couplet and interface stresses

1.0 INTRODUCTION
Masonry which is a composite material made up of brick and mortar elements bonded together definitely would have a stress pattern when subjected to vertical compression load, but up till now there is scarcity of data on its behaviour to stress and especially when you consider the varying elastic properties of the mortar elements. It is as a result of these short-comings that the author has carried out this investigation on the determination of the interface stresses in the brickwork, by use of finite element method. We should note at this point that the splitting failure of brickwork under compressive loads has in general been associated with the transverse strain of the mortar joint due to
the difference in elastic properties of the brick and mortar. Thus by carrying out a finite element elastic analysis of a brick-mortar couplet a relationship can be obtained between the compressive forces and the brick and mortar stresses. This relationship is expected to show how stress in a brick-mortar couplet will influence the overall brick wall structure.

In other related works, attempts were made at a generalized failure theory, which would account for the shear slip and tensile splitting mode of failure [1]. Also papers published [2,3] focused on the study of shear stress path failure criterion for brickwork, where they defined the condition for shear slip mode of failure and also failure in other modes. This is a continuation from previous research work [4] which is on shear strength of unframed brickwalls under the action of compressive force and shear. Again Chinwah, J.G. et al [5] in their publication which is a proceeding of the second international conference on structural Engineering and modeling worked on shake table study of Masonry walls. In this work a possible approach for estimating the shear capacity of a single storey masonry walls for resisting earthquake loads was presented. This study describes the application of a novel earthquake simulation technique (shake table) which was used by the authors for the study of structural responses of unframed single storey masonry wall with brick strength corresponding to the average compressive strength of bricks sampled from thirteen brick industries in Nigeria. We should note at this point that initial research investigation on the in-plane shear behaviour of brick masonry was mainly confined to static test. Experimental studies conducted and reported on masonry structures subjected to stimulated earthquakes were also carried out in 1983 [6]. Another set of researchers in their paper which is also a proceeding of the second international conference on structural Engineering Analysis and Modelling [7,8] worked on the plane of weakness theory for masonry Brick Elements. The paper is an experimental verification of the fundamental behaviour of brick elements subjected to in-plane uniaxial compressive stress forces. In their work they saw brick masonry assemblages as a two-phase composite material. The phases do not only have different strengths but also different deformational characteristics. The mortar phase which acts as a binding medium for the assemblage, introduces two distinct planes of weakness, orthogonal to each other. Also in related work, the behaviour of brick masonry panels in response to uniaxial stress
(compressive) applications was investigated in relation to the plane of weakness theory [9]. The biaxial compressive strength of brick masonry was also studied [10,11].

Apart from work done in [5], further, studies on modeling of masonry structures under dynamic and earthquake loads has also been undertaken and are well published in [12], where the effects of rapid rate of loading using ¼ reduced scale masonry model was investigated.

Brick-mortar elements which are under various stress conditions are primarily vertical load bearing elements in which the resistance to compressive stress is the primary factor in design. Most of the design values at the present have been obtained by empirical basis from test on walls and smaller specimen. This is relatively safe for design as it does not provide meaningful insight into the complex stress nature of brick and mortar elements and as a result it is necessary to postulate a suitable theory of failure for brick wall based on influence of the stress analysis on the brick-mortar couplet on the overall brick wall structure.

In order to achieve the set objectives, this research work shall be limited to the finite element analytical study of brick-mortar couplet subjected to vertical compression. The different elastic properties of the brick and mortar element would be taken into consideration and also from the results obtained the stress distribution of a typical brick-mortar analytical model will be obtained. The influence of the different elastic properties of brick-mortar element on the stress profile would also be seen. This will enable us to determine areas of critical stress states, which most probably may occur at the brick-mortar interface or around the brick.

Hence with an analytical approach like the finite element method the complex stress nature of the brick-mortar couplet can be handled. The finite element method of analysis is a powerful analytical tool which can handle problems resulting from elastic continuum plane stress problems. The finite element formulation of plane stress problems using different types of element shapes has received considerable attention and it is well published. I would close this review by stating that in this part of our world [Nigeria] much attention is not given by engineers to structural brick wall design and construction and this is due to scanty knowledge on the behaviour of brick wall when used as a structural element. In the advance world there had been growing interest in the
design of unframed structural works which has necessitated the development of relevant codes of practice.

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2.0 THEORETICAL ANALYSIS

The aim of this research work is to determine the stress distribution in brick-mortar couplet structure subjected to compression load by finite element method and taking into account the different elastic properties of the brick and mortar component.

In previous work done by researchers, brick wall was regarded as a two-dimensional isotropic plane stress problem, as the different elastic properties of the brick and mortar component were not taking into consideration. The values of modulus of elasticity (E) and prisons Ratio (ν), for brick and mortar would be derived from references from previous works. The finite element method of structural analysis is a method in which a modified structural system consisting of discrete (finite) elements is substituted for the actual continuum and thus represents an approximation, which is of a physical nature. The basic principle of this method is that the continuum is divided into a finite number of elements interconnected at node points situated on their boundaries. The usual shape of plane elements commonly used is triangular and quadrilateral elements. The structure thus idealized can be analyzed by any of the standard method of structural analysis.

For the purpose of this brick-mortar analysis, constant strain triangular elements shall be used and the formulation used is the displacement approach. In using this method the nodal displacements are the basic unknown, while the stresses and strains are assumed constant for each element
By the use of this method the distribution of stress in the block-mortar continuum can be obtained easily, and with the result obtained, areas of critical stress states, which most probably may occur at the brick-mortar interface or around the brick is investigate. This whole process will involve voluminous numerical works which will be considerably simplified by matrix formulation of the whole problem, which is suitable for computerization.

2.1 Derivation of the Triangular Element Stiffness Matrix

The basic steps in the derivation of the element stiffness matrix for a triangular element in plane stress and strain using the displacement approach are well developed. I have only presented here, a brief overview of this approach.

The Cartesian co-ordinate system is shown in Figure 1 and the three nodes of a typical triangular element is numbered 1, 2, 3 using an anticlockwise convention. The positions of these nodes are represented as \((x_1, y_1), (x_2, y_2), (x_3, y_3)\). Considering that all the displacements of the nodes are in the plane, the element has two degrees of freedom at each node, hence a total of six degrees of freedom \((u_1, v_1, u_2, v_2, u_3, v_3)\) for the triangular element. The corresponding forces are \((F_{x1}, F_{y1}, F_{x2}, F_{y2}, F_{x3}, F_{y3})\).

![Figure 1: Nodal forces and displacement displayed in the Cartesian co-ordinate system](image)
Since these vectors contain six terms, the corresponding element stiffness matrix \([K^e]\) for the element would be a 6 x 6 matrix for this plane elasticity triangle. Hence

\[
\{F^e\} = [K^e] \{\partial^e\}
\]  

(1)

In order to choose a displacement function \([f(x, y)]\) that defines displacement \(\{\partial(x, y)\}\) at any point in the element. We consider two linear polynomials

\[
U = \partial_1 + \partial_2 x + \partial_3 y \\
V = \partial_4 + \partial_5 x + \partial_6 y
\]  

(2)

The six unknown coefficients \((\partial_1, \partial_2, \ldots, \partial_6)\) corresponds to the six degree of freedom. Since these displacements are both linear in x and y axis, displacement continuity is ensured along the interface between adjoining elements, for any nodal displacement.

In matrix form equation 2 can be written as

\[
\{\partial(x, y)\} = [f(x, y)] \{\delta\}
\]  

(3)

The state of displacement \(\{\delta(x,y)\}\) within element can be expressed in terms of nodal displacements \(\{\delta^e\}\)

\[
\{\delta(x,y)\} = [F(x,y)] [A]^{-1} \{\delta^e\}
\]  

(4)

where \([A]^{-1}\) is the inversion of the matrix \([A]\) which is a 6 x 6 matrix.

The strains \(\{\varepsilon(x,y)\}\) at any point in the element can be related to displacements \(\{\delta(x,y)\}\) and hence to the nodal displacements \(\{\delta^e\}\).

For plane stress and plain strain problems, the strain vector \(\{\varepsilon(x,y)\}\) is simply represented as

\[
\{\varepsilon(x, y)\} = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}
\]  

(5)

where \(\varepsilon_x\) and \(\varepsilon_y\) are direct strains and \(\gamma_{xy}\) is the shearing strain, and from the theory of elasticity, the following relationship exists between strain \(\varepsilon\) and displacements \(u\) and \(v\).
\[ \varepsilon_x = \frac{\partial u}{\partial x}, \quad \varepsilon_y = \frac{\partial v}{\partial y}, \quad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \]  \tag{6} 

substituting for \( u \) and \( v \) from equation 2 and undertaking the necessary simplifications, we shall arrive as the expression.

\[ \{ \varepsilon(x,y) \} = [B] \{ \delta \} \]  \tag{7}

where the matrix \([B]\) is 3 x 6 displacement transformation matrix, which is a function of the nodal point co-ordinates only.

Relating the internal stresses \( \{ \sigma(x,y) \} \) to strains \( \{ \varepsilon(x,y) \} \) and the nodal displacements \( \{ \delta \} \) for plane elasticity problems, the state of stress \( \{ \sigma(x,y) \} \) at any point may be represented by three components of stress \( \sigma_x, \sigma_y \) and \( \tau_{xy} \).

\[ \{ \sigma(x,y) \} = \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} \]  \tag{8}

where \( \sigma_x \) and \( \sigma_y \) are the direct stresses and \( \tau_{xy} \) is the shearing stress. The stress and strain components are related by the \([D]\) matrix where \( \{ \sigma(x,y) \} = [D] \{ \varepsilon(x,y) \} \). The matrix \( D \) is a 3 x 3 matrix, whose terms depends on whether the problem is one of plane stress or plane strain. Note that for a plane stress problem the value of the stress \( \sigma_z \) which is normal to the plane is zero. The investigation we are carrying out is one of plane stress.

Hence

\[ \{ \sigma(x,y) \} = \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \frac{E}{1-V^2} \begin{bmatrix} 1 & V & 0 \\ V & 1 & 0 \\ 0 & 0 & 1-\frac{V}{2} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \]  \tag{9}
which is represented simply as

$$\{\sigma(x,y)\} = [D] [B] \{\delta^e\}$$  \hspace{1cm} (10)$$

Now replacing internal stress \{\sigma(x,y)\} with statically equivalent nodal forces \{F^e\}, and relating nodal forces to nodal displacement \{\delta^e\} the element stiffness matrix \[K^e\] would be obtained.

When the necessary simplification is done, the final result is represented as follows

$$[F^e] = [[B]^T [D] [B] d(Vol.)] \{\delta^e\}$$  \hspace{1cm} (11)$$

For a plane elasticity problem, \[B\] is a 3 x 6 displacement transformation matrix. The \[D\] matrix depends upon whether the problem being considered is one of plane stress or plane strain.

Form equation 11, the matrix \[B\] and \[D\] contain constant terms and hence can be taken outside the integration, leaving only \[\int d(Vol)\] which, in the case of an element of constant thickness equals the area of the triangle \(\Delta\) multiplied by its thickness \(t\).

Hence if follows that

$$[F^e] = [(B)^T [D] [B] \Delta t] \{\delta^e\}$$  \hspace{1cm} (12)$$

where \(2\Delta = \det \begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix}\)  \hspace{1cm} (13)$$

From equation 12, the element stiffness matrix is obtained simply as

$$[K^e] = [(B)^T [D] [B] \Delta t]$$  \hspace{1cm} (14)$$

Having obtained the element stiffness matrix the nodal displacement is calculated from the nodal forces simply by the relation in equation 1

$$\{F^e\} = [K] \{\delta^e\}$$

This final step is aimed at determining the element stresses from the element nodal displacements. The relationship established in equation 10 enables this process to be carried out, where

$$\{\sigma(x,y)\} = [D] [B] \{\delta^e\}$$

Can be expressed simply as

$$\{\sigma(x,y)\} = [H] \{\delta^e\}$$  \hspace{1cm} (15)$$
3.0 STRUCTURAL MODEL

The analytical brick-mortar model shall consist of the full size brick and mortar couplet. The couplet shall consist of brick panels of average dimensions 416mm x 371mm x 110mm, fabricated from high quality perforated bricks of standard dimensions. The physical and elastic properties of brick and mortar component are given in table 1 and 2.

Table 1: Physical properties of brick and mortar

<table>
<thead>
<tr>
<th></th>
<th>Brick</th>
<th>Mortar</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (mm)</td>
<td>416mm</td>
<td></td>
</tr>
<tr>
<td>Breadth (mm)</td>
<td>371mm</td>
<td></td>
</tr>
<tr>
<td>Thickness</td>
<td>110mm</td>
<td></td>
</tr>
<tr>
<td>Thickness</td>
<td>19mm.</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Average values for Modulus of Elasticity and Poissons Ratio

<table>
<thead>
<tr>
<th></th>
<th>E(N/mm) x 10^3</th>
<th>ν</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brick</td>
<td>8.83</td>
<td>0.060</td>
</tr>
<tr>
<td>Mortar</td>
<td>See comment*</td>
<td>0.170</td>
</tr>
</tbody>
</table>

*Comment:- The value of modulus of elasticity E for mortar varies with the actual water/cement ratio or cube crushing strength f_{cu} while the Poissons ratio ν for mortar can be taken as a average of 0.170. The value for E_m from literature varies from 1.24 x 10^3 N/mm^2 to about 37.23 x 10^3 N/mm^2.

Note that the values for elastic properties seen in the table above are as a result of empirical values obtained from compression tests on brick-mortar model and brickwork (wallets).

The following assumptions are made in this analysis in order to formulate a finite element model for the analysis of the stress in the brick-mortar couplet as typically resented in figure 2.

(i) The size of the brick model would be taken as 416mm x 371mm x 110mm.
(ii) The thickness of the mortar joint would be taken as 19mm.
(iii) For loading condition of the analytical brick-mortar couplet, we assume that a compressive force consisting of a unit of uniformly distributed load (udl) acts on it.

(iv) The support reaction of the couplet will consist of a fixed support on one end and a roller support on the other end and this is to ensure a determinate structure.

The brick-mortar couplet is analysed as a plane stress problem using finite element method as an analytical tool.

A typical representation of a brick-mortar couplet under uniform loading is shown in figure 2

Two types of analytical models are shown in figures 3 and 4. While model 1 is divided into 100 triangular elements, model 2 is a more typical representation of a practical couplet with 216 elements. It is also interesting to note that the elements within the mortar joint are made smaller in two rows. The idea in the smaller mesh size seen in model 2 is to as much as possible keep the aspect ratio as near unity as possible, i.e. to avoid long thin triangles since their use reduces accuracy. It is always necessary to have several idealizations to check if the results converges as the number of elements is increased.

Figure 2: Typical representation of a brick-mortar couplet under uniform loading
Figure 3: Analytical model 1 ready for finite element analysis
Figure 4: Analytical model 2 ready for finite element analysis

371mm Brick

19mm Mortar

371mm Brick
4.0 COMPUTER ALGORITHM

The basic steps to obtain the element stiffness matrix \([K_e]\) and stress matrix \([H]\) have already been discussed in details. A finite element solution will therefore involve the calculation of the respective stiffness matrix for every element in the idealized structure and then assembling them to obtain the Global matrix otherwise known as the structural stiffness matrix \([K]\). The main objective in this work will be to determine the nodal displacement which in turn would be used to determine the stress in each element, from which the stress pattern for the overall structure can be evaluated. However, this whole process involves voluminous numerical works which is simplified by the use of reliable computer programme for finite element analysis. Hence an electronic computer with good storage capacity would be very essential for this work.

The finite element code for this work shall be developed using the visual basic version 6.0 programming language, built up in a number of subroutines, each of which takes care of basic steps in the overall analysis.

For the purpose of deriving a computer code, the basic general step in for the derivation of the element stiffness matrix \([K_e]\) was not followed as in coding a proper computer programme certain process has to be skipped or readapted. Hence there are adapted steps to follow in order to obtain a computerized solution for the analysis.

However, the basic requirement of the computer program necessary for the complete solution of a problem by the finite element method involves using the input data which describes fully the idealized structure and its loading and in turn produces output consisting of tabulated nodal displacements and element stresses. The input data consists of specifying the geometry of the idealized structure, its material properties, the loading and how it is supported in space. The data also includes certain control numbers that may help the generality and efficiency of the program and should be supplied easily in the input data, such as the total number of nodes and elements and this helps the routine to determine how much storage is required.
5.0 DISCUSSION OF RESULTS

The results of the finite element analysis can be displayed to show the effect of the load (unit udl) on the brick-mortar couplet continuum, taking into consideration the difference in the elastic properties of the brick and mortar component.

Note also that because of the varying values of elastic modulus for mortar ($E_m$) the analysis for the brick-mortar model was carried out to show how the varying values of $E_m$ can affect the results obtained while keeping other parameters constant. Hence three different values for $E_m$ were used producing 3 different sets of results for each model. It is also pertinent to state that the results for the different models are consistent and converging as the number of elements is increased. It is also necessary to note that results from finite element solution under-estimate the exact answers but as the finite element subdivision is increased the results approach the exact solution.

Interface Stresses

The stresses at the brick-mortar interface are one area to concentrate on because it is at this point we notice the effect of the different elastic properties. A look at results show the different component of stresses ($\sigma_x$, $\sigma_y$ and $\tau_{xy}$) much reduced at the interface area, (i.e. area around the brick-mortar joint) when $E_m$ is low. Similar results are also seen as $E_m$ is increased, but at very high values of $E_m$ ($E_m = 31.03, 37.23$) the interface stresses are seen to be higher compared to the brick region.

The values for displacement at this interface sections show relatively high values for both the normal and lateral displacements, when you consider small values for $E_m$. The positive and negative values of stress around this area show compressive and tensile stresses around this area. Hence a graphical representation can be done to show the stress mechanisms around this interface region. This may be done by picking the highest value for stress in each column of the analytical model.

It is easily seen that failure occurs usually at the brick-mortar interface when the value of $E$ for mortar is relatively low. Hence the usual theory for brickwork that failure occurs generally at the brick-mortar interface is questionable as from my result there is a limit.
The lateral stresses are predominantly tensile in the brick region but compressive in the mortar region near the interface as the reason is easily traced to the different elastic properties the region possesses.

The shear stresses ($\tau_{xy}$) at the interface region is relatively smaller than the normal and lateral stresses ($\sigma_x$, and $\sigma_y$) and so it can be deduced that failure around this region is predominantly caused by the shear stresses.

Tensile mode of failure normally occurs at the interface when the E value for mortar is relatively high. Hence lateral stress which is the major cause of failure is tensile in nature.

**General Tensile and Compressive Failure**

A look at the tables shows tensile and compressive stresses occurring with the minimum values showing where tensile or compressive split is likely to occur. These splits generally extend diagonally from the brick region crossing the mortar joints, and this depends on the strength of the mortar joints which is dependent on the elastic modulus value.

From the observation of the tables it is pertinent to note that the tensile/compressive failure modes around the brick-mortar couplet is a complex one which depends on the complex state of stress induced and hence to obtain a general pattern for tensile and compressive stress a more concise study of failure under varying biaxial load system would be considered.

**Interface Principal Stresses**

The principal stress $\sigma_1$, $\sigma_2$ and the maximum shear stress $\tau_{\text{max}}$ when considered around the interface shows that, the principal stress $\sigma_1$ is mainly tensile (+ve value of stress) while $\sigma_2$ is predominantly compressive (-ve value of stress). The importance of the principal stresses lies in the fact that they are the maximum and minimum values of the normal stresses and when they are opposite each other they give the numerical values of the maximum tensile and compressive stresses and this normally occurs when the shear stress ($\tau_{\text{max}}$) occurs at planes 45\(^\circ\) to the principal planes. Hence it can be deduced from the finite element results that failure around the brick or mortar close to the interface are mainly tensile in nature which is the direct cause of the principal tensile
stress $\sigma_1$, it is also seen that the principal stress $\sigma_2$ which is predominantly compressive in nature is also seen around the interface region. We should note at this point that higher values of $\sigma_1$, and $\sigma_2$ indicates failure around the brick mortar couplet for varying values of elastic modulus for mortar ($E_m$).

**Critical Interface Tensile Stress**

Result from the finite element analysis shows that the principal stresses $\sigma_1$ adjacent to the interface is identical both in value and distribution to the lateral stress $\sigma_x$. The angle ($\delta$) which is its inclination to the X-axis is approximately zero throughout, hence $\sigma_x$ and $\sigma_1$ are also identical in direction. Thus it can be seen that tensile failure initiated in the brick or mortar adjacent to the interface may be attributed to principal tensile stress $\sigma_1$.

The mode of failure is usually by vertical splits and this can easily be seen in a brick-mortar couplet failure indicating two main vertical splits occurring in positions corresponding to be position of peak tensile stress. These vertical splits are initiated by relative lateral deformation of each material at the interface. This relative movement and hence the lateral stress is a function of $v/E$ for both materials. It relationship can be established by plotting the critical or maximum value of the lateral stress ($\sigma_x$) adjacent to the interface against an elastic property parameter $x$, where

$$\sigma_{x(max)} = f(x)$$  \hfill (16)

where

$$x = \frac{\varepsilon_1}{\varepsilon_2}$$

$$\varepsilon_1 = \frac{v_1}{E_1} \quad \text{(for brick)}$$

$$\varepsilon_2 = \frac{v_2}{E_2} \quad \text{(for mortar)}$$

Hence two curves are obtained, one for maximum $\sigma_x$ in brick, and the other for maximum $\sigma_x$ in mortar (figure 5). For both curves where $\sigma_x = 0$, $x = 1$. In other words, the lateral stress (and hence the relative lateral strain) in the brick and mortar adjacent to the interface is zero when $v_1/E_1 = v_2/E_2$. This result can be shown to be in good agreement with elastic theory as follows:
The interface lateral strain is given by

\[ \varepsilon_x = \frac{1}{E} \left( \sigma_x - \nu \sigma_y \right) \]  

(17)

From which the relative interface lateral strain may be written as

\[ \varepsilon_{x1} - \varepsilon_{x2} = \frac{1}{E} \left( \sigma_{x1} - \nu_1 \sigma_{y1} \right) - \frac{1}{E} \left( \sigma_{x2} - \nu_2 \sigma_{y2} \right) \]  

(18)

At the interface \( \sigma_{y1} = \sigma_{y2} = \sigma_y \)

Hence for \( \sigma_{x1} = \sigma_{x2} = 0 \)

\[ \varepsilon_{x1} - \varepsilon_{x2} = \sigma_y \left( \frac{\nu_2}{E_2} - \frac{\nu_1}{E_1} \right) \]  

(19)

It is thus obvious that the relative strain \( \varepsilon_{x1} - \varepsilon_{x1} \) and also the lateral stress \( (\sigma_x) \) are zero, when \( \frac{\nu_1}{E_1} = \frac{\nu_2}{E_2} \). The above result illustrates the controlling effect of this parameter \( \nu/E \) on the interface lateral stress.

The two curves showing the variation of the maximum interface lateral stress \( \sigma \) with the parameter \( x \) may be represented by the following polynomial functions fitted through the values obtained by finite element analysis.

\[ \sigma_{x(max)} = \left( a_0 + a_1 x + a_2 x^2 + a_3 x^3 \right) p \]  

(20)

where \( x \) is as defined above.

\( a_0, a_1, a_2, a_3 \) are constants for the range of elastic properties of the brick and mortar considered and have different values for the two curves as shown in figure 5. 

\(( p)\) is the applied compressive stress and note that lateral stress \( (\sigma_x) \) is a percentage of the applied compressive stress \( (p) \).
6.0 CONCLUSIONS

A finite element modeling of stress distribution in a brick-mortar couplet was carried out. Results were obtained from the finite element analysis which displayed the normal, lateral and shear stresses around the continuum and also the principal stresses. From the results obtained we arrive at the following conclusions:

(i) The failure of a brick-mortar couplet subjected to compressive loads is controlled by the resulting slate of stress within the couplet.

(ii) The behaviour of brick-mortar couplet to stress is easily seen through the stress profiles.

(iii) Three modes of failures are seen from the stress pattern as follows:

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Figure 5: Relationship between critical value of the lateral stress $\sigma_x$ against elastic property parameter $X$
Shear slip mode of failure at the interface region as a result of the shear stresses which is prevalent mostly when E value for mortar is relatively low.

Interface boundary tensile failure in form of diagonal cracks extending from the interface region into the brick region which also is dependent on the elastic modulus value for mortar and bricks.

A very complex tensile/compressive mode of failure which generally extends diagonally from the brick region into the mortar region and depends largely on the strength of the mortar joints which depend on the E value of mortar.

The state of stress at different elemental units of the brick-mortar couplet is thus investigated.

7.0 RECOMMENDATIONS

(i) The stress pattern as seen in the brick-mortar couplet can be used to make a meaningful investigation into the stress and displacement pattern in a brick wall especially when we put into consideration the different elastic properties of the brick and mortar.

(ii) The computer code obtained for the analysis of this model can also serve as a useful tool to make quick investigation into the stress of a brick wall by easily putting into analysis a brick-mortar couplet of the brickwork especially when you consider the unsteady elastic properties of brick and mortar.

(iii) The test result can be used to deduce a permissible stress recommendation for the code of practice for brickwork.

(iv) Logical suggestions for further research are as follows:

- To investigate the stress profile under increased loads.
- A general study of brick-mortar couplet under varying biaxial load system is required to establish a general model for failure.
- A study of the effects of dynamic loads like earthquake forces on the brick-mortar couplet is also necessary.
REFERENCES


