INVESTIGATION OF THE STRAIN ENERGY DENSITY AROUND OPENING IN CONCRETE WALL
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Abstract - The paper presents an analytical study using the finite element method of the strain energy density in planar problem. In this work investigated the most intense zones in terms of the strain energy density in the elastic deformation stage of a concrete wall. The effect of the openings in the concrete wall and comparison of the strain energy density in the solid wall and the wall with openings is studied. And also, it was determined the influence of the size and shape of the openings on the distribution of the strain energy density in the concrete wall. 

Key words: concrete wall, finite element method, strain energy density.

1. INTRODUCTION

The aim of this work is an approach to energy gradients in concrete wall, using the distribution of strain energy density (SED).

The strain energy as a control parameter allows the engineer, on the one hand, a more careful approach to the dangerous section of the walls and accurately finds the point of stress concentration; on the other hand, it allows managing the stress state of the structures. Known, that the ideal in terms of energy distribution is the isoenergetical (equally energetically) design, where the strain energy density at all points of the structure is equally. At first glance, look like an impossible task, but given the fact that we can manage the physics and geometry of the structures, we can try to reduce the accumulation of energy in sensitive places such perimeter of the openings in the concrete or masonry walls. Why, in this paper the density of strain energy in a certain section of the solid wall and wall with different sizes of openings is investigated, and presented its qualitative and quantitative analysis.

2. THEORY AND RESOLVING EQUATIONS

We know that a fundamental concept for the solution of the basic problems in mechanics is based on the principle of conservation of energy.

Under the influence of external forces, the elastic body experiences some deformations, in which the forces make some work. This work is transformed into potential energy and subsequently, removing the external forces, this energy restores the body to the original shape. Energy accumulated during deformation in the unit volume of material, selected around a given point, called the specific potential energy [1, 2].

In the case of multiaxial random loading, a generalized energy criterion based on the energy density parameter of normal and shear strains acting in the critical plane was applied. This criterion was used for determination of the equivalent strain energy density parameter, reducing the multiaxial stress state to the uniaxial one. The paper also contains a review of models for determination of non-local stresses and strains under stress gradients in the material. Next, an equation for calculation of the non-local equivalent strain energy density parameter in the critical plane was proposed [3].

When body is subjected to the multiaxial (general state) stress, the strain energy associated with each of the normal and shear stress components [4]:

\[
U_l = \int_V \left[ \frac{1}{2} \sigma_{xx} \varepsilon_{xx} + \frac{1}{2} \sigma_{yy} \varepsilon_{yy} + \frac{1}{2} \sigma_{zz} \varepsilon_{zz} + \sigma_{xy} \varepsilon_{xy} + \sigma_{yz} \varepsilon_{yz} + \frac{1}{2} \tau_{x} \varepsilon_{x} \right] \, dV \quad (1)
\]

We know that in plane stress-strain state there are only \(\sigma_x, \sigma_y\), and \(\tau_{xy}\) (Fig.1), the potential energy each is as follow:

![Stress state of the 2D element of the concrete wall.](image)

Fig.1. Stress state of the 2D element of the concrete wall.

If applied only \(\sigma_x\)

\[
\langle U_l \rangle_1 = \int_V \frac{\sigma_x^2}{2E} \, dV = \frac{\sigma_x^2}{2E} V \quad (2)
\]

When \(\sigma_y\) is applied in the second stage, the normal \(\varepsilon_x = -\nu \sigma_y / E\). therefore the strain energy for the second stage is:

\[
\langle U_l \rangle_2 = \int_V \left( \frac{\sigma_y^2}{2E} + \sigma_y \varepsilon_{xx} \right) \, dV = \int_V \left[ \frac{\sigma_y^2}{2E} + \sigma_y \left( -\frac{\varepsilon_{xx}}{E} \right) \right] \, dV \quad (3)
\]

Since \(\sigma_x, \sigma_y\) are constants,

\[
\langle U_l \rangle_2 = \frac{V}{2E} \left( \sigma_x^2 - 2\nu \sigma_x \sigma_y \right) \quad (4)
\]
Strain energy due to shear stress:

\[ (U_2)_s = \frac{1}{2\mu} \int \frac{V^2}{2G} dV = \frac{V^2}{2G} \tag{5} \]

The total strain energy is

\[ U_1 = (U_1)_s + (U_2)_s + (U_3)_s = \frac{V}{2E} (\sigma_x^2 + \sigma_y^2 - 2\nu \sigma_x \sigma_y) + \frac{V^2}{2G} \tag{6} \]

And the SED for 2D (plane stress) problem is:

\[ \frac{U_1}{V} = \frac{1}{2E} (\sigma_x^2 + \sigma_y^2 - 2\nu \sigma_x \sigma_y) + \frac{V^2}{2G} \tag{7} \]

3. EXAMPLE, RESULTS AND DISCUSSION

To illustrate the distribution of the SED let us consider a solid concrete isotropic wall and four other walls with different size of opening as shown in figures of the table1. In this study we used the theoretical finite element method (FEM) for determination of the SED in the vicinity of openings.

All investigated problems in the present work were modeled using COMSOL program and Lagrange solid elements. Material properties: stiffness \( E \) of 25GPa and Poisson’s ratio \( \nu \) of 0.33. All presented models are loaded only in the Y-axis direction, with constant the thickness dimension oriented with the model’s z-axis (Thickness=10cm). The model’s x-axis therefore describes the width dimension, perpendicular to the loading direction. The problems investigated here vary in area and shape of hole. (Fig.2)

\[ q=15kN/m \]

Concrete wall, \( E=25e9Pa, \nu=0.33 \)
\( \alpha=10e-6 \quad 1/K, \rho=2300kg/m^3, H=3m \)
\( L=5m, \text{thickness}=0.1m \)

Fig.2. Investigated concrete wall loading, supporting, and properties in COMSOL environment.

At the first step of solution search is made for the law of rectangular solid cross-section height change \( h(x) \) at its constant width \( b=\text{const}. \) Then from the equation conditions of rectangular and hollow cross-section integral characteristics box cross-section constructive parameters are determined.

However, when the wall contains discontinuity, such as shown openings (Table1) sudden change in cross section, high localized SED may also occur near the discontinuity, which coincides with the stress concentration zones \([5, 6]\). As shown in table1, for wall with openings, high SED distribution will be at the chord along upper chord of opening.

The described FEM model was verified by comparing the results of stress along Y-axis for 5 types of wall, first from which is the solid concrete wall (without opening) and others are with opening, as shown in table1. Thus, the comparison between SED for wall with opening was done.

As we can see from figures of the table1 the location of the maximum SED for wall without opening is along the base of the wall, but for wall with opening is around opening. The presented in table1 results show how the opening size affects on the distribution of the SED.

As noted above, the shape of the opening significantly, exerts on the kind and value of the SED (see results in table1). The ratio of the maximum SED (in the wall with openings) to the nominal SED (in the solid wall) at the same section (see dotted lines) is denoted by \( K \):

\[ K = \frac{\text{SED}_{\text{max}}}{\text{SED}_{\text{nominal}}} \tag{8} \]

As we can see, when the size of the opening increases, SED around openings increases, consequently the, \( K \), increases.

This research provides an overview for the design of the required reinforcement in the location of maximum SED at all sides of the opening. Design and placement of reinforcement for wall with openings, particularly, described in Design of Rectangular Openings in Precast walls Under Combined Vertical and Lateral Loads.

The concentration of SED acting around openings is not uniform, easily; we can observe that the SED at corners of the opening reaches its maximum magnitude.

Thus, knowing the SED concentration zones, we control and manage stress state of the wall and reduce the effect of the high stress in these zones. That ultimately lead to the optimization of the structure and as consequence to economy of the materials.

Unfortunately research in this area is very little work, so the comparison is done only with analytical methods of the equation are given above. Seeing workload and evidence of analytical computing this process is not included in the work.
<table>
<thead>
<tr>
<th>Wall Scheme</th>
<th>Diagram of distribution of strain energy distribution along L/2 at dotted level of wall</th>
<th>$K = \frac{\text{SED}<em>{\text{max}}}{\text{SED}</em>{\text{nom}}}$</th>
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4. CONCLUSION

1. The finite element model is used to conduct a SED investigation in concrete wall.
2. The parameters of studied are the opening size and shape.
3. The variability investigation is studied to identify the SED.
4. The results indicate that the maximum SED are around opening.
5. The presence of openings can affect the location and magnitude of the maximum SED around openings.
6. Using a computer program COMSOL allows more accurate approach to the problem of SED.
7. This study shows that the size of the openings has a significant effect on K.
8. Given that the main problem against the engineer is to know the value of SED at any point in structure, and since this work gives a complete picture of the state of SED in each zone of the wall, leads us to a more efficient use of the material, but also helps control the shape of the openings and their position. As a consequence, leads to the economic feasibility of the structural analysis and design.
9. Assessment of duration and definition of behavior of the wall with crack around opening are very important issues in structural analysis and design. The calculation of the SED, the crack extension and service life of structural elements require a lot of attention of designers. In this contact, further work in this direction will help designers more accurately represent the stress state of the walls. This study can help us intuitively and easily find stress concentration zones and true solution.
10. The purpose of this study was to confirm the accuracy of empirical models and methods based on experimental data. In this connection, I must say that the accuracy and calculation speed by FE modeling is much superior to classical methods.

5. REFERENCES

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