Plane Turbulent Wall Jets in Limited Tailwater Depth

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***Abstract*— This paper presents laboratory study of plane turbulent wall jets with finite tailwater depth. The main objective was to show that there is a distinct difference between the behavior of wall jets with relatively shallow and those with relatively deep tailwater depth ratios. A set of six experiments, with tailwater depth ratios varying from 5.3 to 20, were conducted to observe and quantify the growth of the wall jet, the decay of the velocity scale and the variation of the volume flux, and momentum flux. The results from these experiments were compared with previous studies with relatively deeper tailwater depth ratios. This study contributes to an understanding of the behavior of plane turbulent wall jets when the ambient fluid has a limited extent.**

KEYWORDS— Turbulent flow, Wall jets, Tailwater.

I. INTRODUCTION

A classical wall jet, as defined by [12], is a jet of thickness b_o and uniform velocity U_o issuing from a rectangular slot tangentially to a flat plate submerged in a semi-finite expanse of the fluid (see Fig. 1). As soon as the jet leaves the slot, a shear layer develops on the fluid side while a boundary layer develops on the wall side. The section at which the two layers meet is the end of the potential core and downstream of this section the flow is developed. Experimental observations of the variation of the axial velocity u(y) with y, at different x-stations, showed that the velocity profiles have the same shape at the different stations. The data of [6] showed that, at any x-station, the axial velocity u increases from zero at the wall to reach its maximum value u_m at a distance of $y = \delta$ and then decreases to reach zero at large y. The boundary layer is the region between $y = 0$ and $y = \delta$. The region above the boundary layer is known as the free-mixing region. To check for the similarity of the velocity profiles, the maximum velocity u_m, was chosen as the velocity scale and the length scale b was taken as the distance, above the flume bed, at which the axial velocity $u = 0.5u_m$ and $\partial u/\partial y$ is negative. Using the experimental observations of [6], [12] found that the velocity profiles were similar.

For turbulent jets discharged from slots or orifices in walls into large ambients at rest, it has been generally assumed that the momentum flux will be preserved [1], [12], and [14]. In some experimental investigations [7], [8], [9] and [11], it was observed that the momentum flux decayed to some extent at large distances from the origin of the jets. [10] observed that the momentum flux could decay to about 80% of that at the source, at a longitudinal distance equal to 100 b_o for plane turbulent jets, where b_o is the slot width. [10] attributed this loss of momentum flux to the negative momentum carried by the entrained fluid which approached the jet at an angle of about $\pi/4$ rad. from the forward direction of the jet. Based on an approximate integral analysis, [10] developed an equation to describe the variation of the momentum flux with the longitudinal distance from the nozzle producing the jet. [15] attempted to explain this decay of momentum flux by combining analysis of the jet with the flow in the region surrounding the jet. [15] coupled the jet and the induced outer flow through momentum and volume balances.

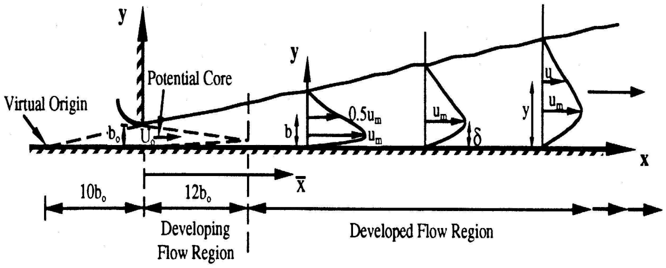

Fig. 1 Definition sketch of plane turbulent wall jet (Ead and Rajaratnam 1999)

[16] studied the variation of momentum and volume fluxes as well as the growth of plane turbulent surface jets with limited depth of tailwater. [16] conducted 10 experiments to study the effect of the finite depth of tailwater on the characteristics of the surface jet and to observe the variation of the momentum and volume fluxes and the breakdown of the surface jet due to the limited depth of the ambient. [16] also used the experimental results of [17] and of [13]. Their results showed a momentum decay and a breakdown (or variation from that of jets in infinite ambient) in the velocity and length scales due to the jet confinement. Recent investigations by [2] and [4] have shown that the decay of the momentum flux is significant even for relatively large tailwater over a distance of 300–900 slot widths.

[5] investigated, theoretically and experimentally, plane turbulent wall jets with finite depth of tailwater. Froude

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number used was in the range of 4 to 8 and the tailwater depth ratio ranged between 25 and 50. It was observed that the momentum flux of the forward flow in the wall jet decays appreciably with the distance from the nozzle. [5] mentioned that the momentum decay was due to the entrainment of the return flow, which had negative momentum that required a depression of the water surface near the gate housing the slot. The growth of the wall jet, the decay of the velocity scale, the variation of the volume flux, momentum flux, and the length of the surface eddy were also studied by [5].

This study contributes to an understanding of the behavior of plane turbulent wall jets in limited tailwater depth. It presents the results of an experimental investigation of plane turbulent wall jets with tailwater depth ratios equal to 20 or less. It was thought it would be interesting and useful to compare the behavior of such jets with relatively shallower tailwater depth ratios, with past studies with relatively deeper tailwater depth ratios.

II. EXPERIMENTAL SETUP AND EXPERIMENTS

Experiments were conducted in a rectangular flume of the re-circulating type, 0.15 m width, 0.30 m depth and 2.45 m length, with 13 mm thickness clear Plexiglas sides and a bed made of steel covered with plastic sheets. All experiments were performed on a horizontal bed. A single-leaf head gate with a streamlined lip was used to produce a supercritical stream with a thickness equal to the gate opening, b_o. A tailgate, installed at the downstream end of the flume, was used to control the tailwater depth. The head tank was 0.332 m length, 0.150 m width and 0.765 m depth, with Plexiglas sides. A metal tank, located under the flume, 0.30 m width, 2.45 m length and of variable depth from 0.47 to 0.67 m, was used to collect outflow and a centrifugal pump discharged it back into the flume. Two valves were used to control the discharge.

Six experiments were conducted and the primary details of these experiments are shown in Table 1. Two experiments with tailwater depth ratio equal to 50, from [5], are also included in Table 1. A Prandtl tube with an external diameter of 3.0 mm was used to measure the time-averaged longitudinal velocity u along vertical sections at different longitudinal distances from the opening producing the jet. The Prandtl tube was connected to a manometer board fixed on the side wall of the flume. All the measurements were taken in the middle third of the flume. In these experiments, the tailwater depth ratio η was in the range of 5.3-20.0 and the Froude number was in the range of 3.0-8.0. The Reynolds number of the jet was in the range of 9,500–25,200.

III. EXPERIMENTAL RESULTS

Fig. 2 shows a typical velocity field for a wall jet in shallow tailwater depth (Expt. 4) along with the water surface profile. Fig. 2 shows the velocity profiles only in the forward flow as the tailwater depth is relatively shallow and the surface eddy is quite limited. Velocity profiles are drawn for x/b_o varying from 8 to 100. In Fig. 2, y is the vertical distance above the bed. Also, Fig. 2 shows a drop in the water surface in the vicinity of the gate for x up to 30 cm followed by a rise in the water surface for x in the range of 30-60 cm. It is understood that the depression in the water surface profile and hence the water surface back slope are responsible for the reverse flow.

Table 1. Primary Details of Experiments

Expt. (1)	b_o (cm) (2)	U_o (m/s) (3)	F_o (4)	y_t (cm) (5)	$\eta = y_t/b_o$ (6)	R_n (7)
1	1.0	0.95	3.0	5.3	5.3	9500
2	1.0	1.24	4.0	20.0	20.0	12400
3	1.0	1.25	4.0	50.0	50.0*	12500
4	1.0	1.88	6.0	10.0	10.0	18800
5	1.0	1.88	6.0	15.0	15.0	18800
6	1.0	1.88	6.0	20.0	20.0	18800
7	1.0	2.52	8.0	20.0	20.0	25200
8	1.0	2.50	8.0	50.0	50.0*	25000

*[5]

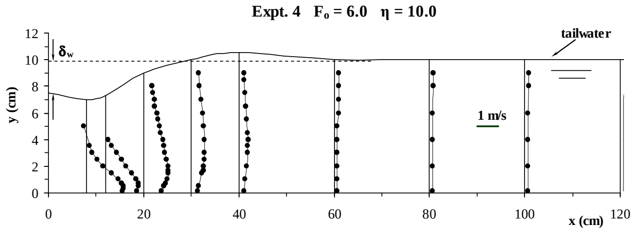

Fig. 2
Velocity fields of wall jets in limited tailwater (η = 10.0)

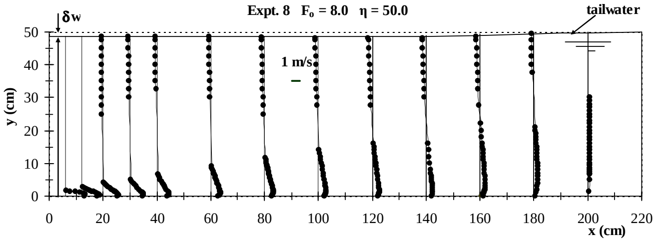

Fig. 3
Velocity fields of wall jets in limited tailwater (η = 50.0)

Fig. 3 shows velocity profiles for Expt. 8 [5]. In this experiment the tailwater depth ratio η is 50 and the relatively deep tailwater allowed a much longer and deeper eddy and the reverse flow could be captured for a distance x up to 200 cm. It can easily be seen that the wall jet in the relatively deeper flow is well-defined for x up to 200 cm while in the shallower flow the wall jet could be observed for x up to only 40 cm. Also, it is clear that the wall jet expands much faster in the shallower flow than in the deeper one. These differences showed that the conclusions of [5] for wall jet in shallow tailwater should be revised for wall jets in relatively shallower tailwater.

To check for the similarity of the forward flow velocity profiles, the maximum velocity u_m at any x-station was selected as the velocity scale while the length b was selected as the length scale where b is equal to the vertical distance above the bed y where u is equal to half the maximum velocity and $\partial u/\partial y$ is negative (see Fig. 1).

Typical velocity profiles in the fully developed flow region ($x/b_o \geq 12$) are shown in Figs. 4 and 5. Fig. 4 is for Expt. 4 (F_o = 6.0 and η = 10.0) and Fig. 5 is for Expt. 8 (F_o = 8.0 and η = 50.0). Figs. 4(a) and 5(a) clearly show the wall jet features. Figs. 4(b) and 5(b) show a similarity profile with u_m and b chosen as the velocity and length scales, respectively. In Fig. 5, although some velocity profiles were taken for $x/b_o > 140$, these profiles were excluded as they were somewhat different from those shown. A comparison between the two figures, Figs. 4 and 5, shows that for the greater tailwater depth ratio (η = 50.0) the velocity profiles show better similarity and conform more with the classical wall jet than the shallower depth ratio (η = 10.0).

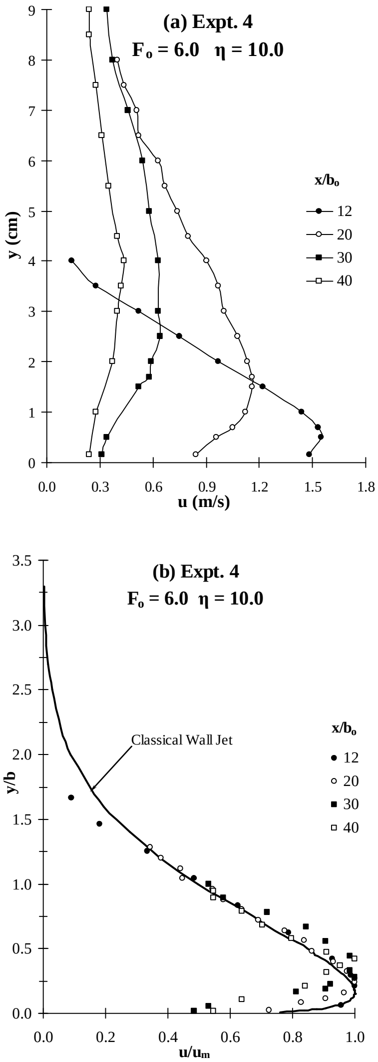

Fig. 4 Velocity distribution in the developed flow ($x/b_o \geq 12$)
(a) Typical velocity profiles (Expt. 4);
(b) Similarity profile (Expt. 4)

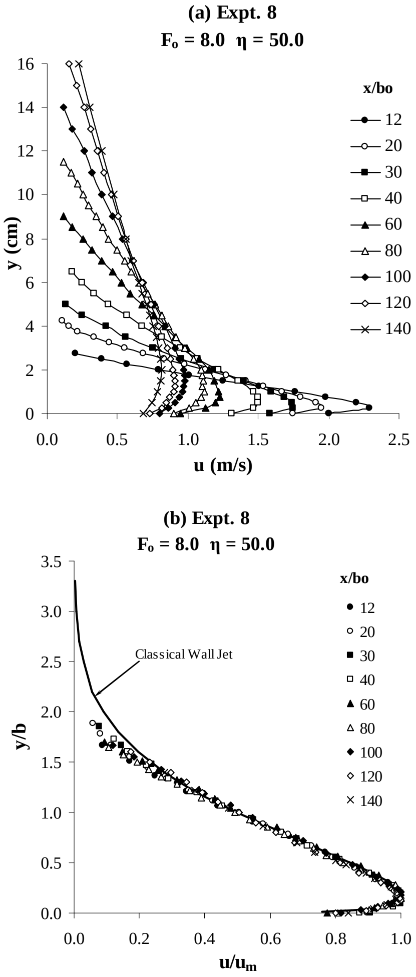

Fig. 5 Velocity distribution in the developed flow ($x/b_0 \geq 12$)
(a) Typical velocity profiles (Expt. 8);
(b) Similarity profile (Expt. 8)

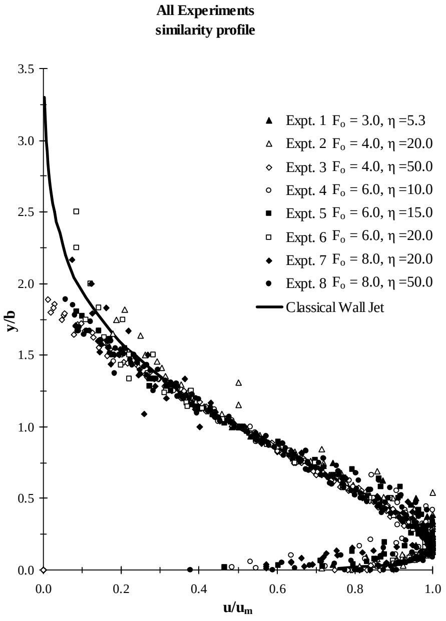

Fig. 6 Consolidated non-dimensional graph for the velocity distribution in the wall jet

Fig. 6 shows a consolidated plot of all the data for the eight experiments. A study of the profiles shown here established that, except for the far-field velocity profiles (see Fig. 3), the velocity distributions are similar. The similarity profile follows that of the classical plane turbulent wall jet except near the upper end of the profile, where it shows a linear distribution. This deviation between the similarity profile of the present data and that of the classical wall jet profile is possibly due to the interaction between the forward and backward flows. This interaction is more profound for the shallower tailwater depth ratios and gives a greater deviation from the classical wall jet.

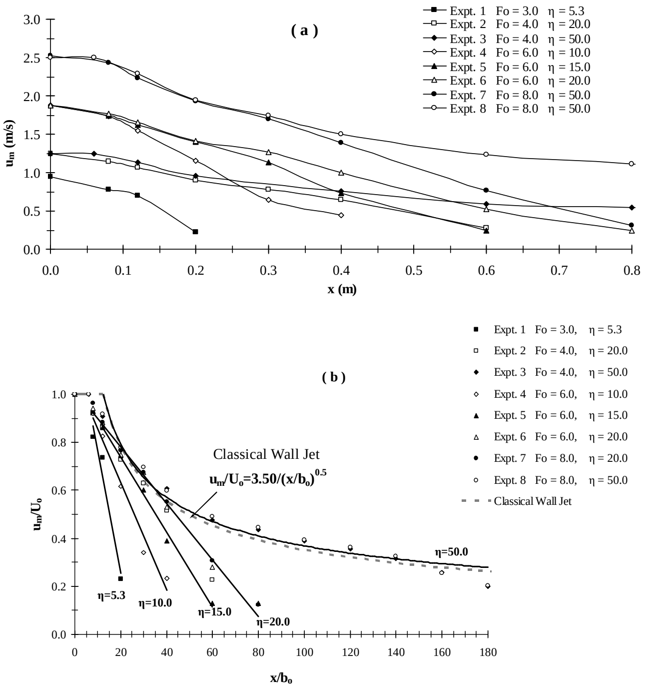

Fig. 7 Variation of the maximum jet velocity with distance

Having found that the velocity profiles in the forward flow are similar, it is necessary to study the variation of the velocity scale u_m and the length scale b with the distance x. Fig. 7(a) shows the decay of the maximum velocity with the longitudinal distance from the slot. The maximum velocity at the slot was varied from 0.95 to 2.52 m/s and u_m was measured for x up to about 0.8 m. Fig. 7(b) shows the decay of the maximum velocity u_m at any section in terms of the velocity of the jet at the slot, U_o, with the normalized distance from the gate x/b_o. The decay of the maximum velocity for the classical wall jet is also plotted for comparison. Figure 7(b) shows a clear distinction between the decay of the maximum velocity for the shallow (η = 5.3, 10.0, 15.0 & 20.0). and the deep (η = 50.0) tailwater depth ratios. For shallower tailwater depth ratios, the decay of the maximum velocity is rapid and the variation of u_m/U_o with the normalized distance x/b_o is almost linear. For the deep tailwater depth ratios the maximum velocity decays slowly and the variation of the u_m/U_o with the normalized distance x/b_o follows that of the classical wall jet up to x/b_o is equal to about 160 and then deviates to show a faster decay. Fig. 7(b) also indicates that the Froude number does not have a significant effect on the velocity decay.

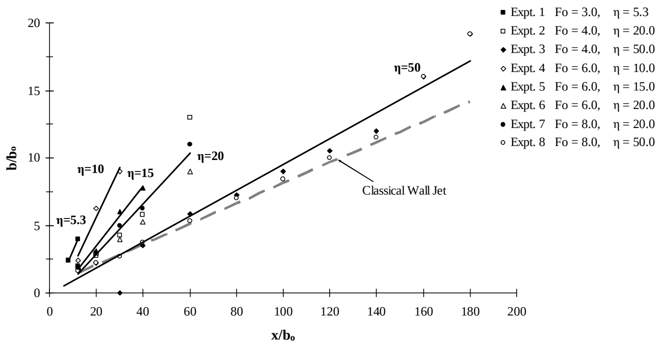

Fig. 8 Variation of the jet half width with distance

The growth of the length scale of the jet (or the half-width b) with distance is shown in Fig. 8. This figure also confirms the clear distinction between the behavior of the shallow and the deep wall jets. The growth of the jet half-width with distance for the shallow tailwater depth ratios (η = 5.3, 10.0, 15.0 & 20.0) is rapid while that for the deep tailwater depth ratio (η = 50.0) is slower and conforms more to the classical wall jet relation. Further, it is found that the Froude number does not have any significant effect on the growth of the jet half-width.

For all the experiments, at every section where velocity observations were made, the forward flow rate Q and the forward momentum flux M per unit width were calculated as the sum of the fluxes through thin strips. Fig. 9 shows the variation of the relative discharge Q/Q_o in the wall jet with distance x/b_o (where Q_o is the discharge at the slot). A study of Fig. 9 shows that, the behavior of jets with shallow tailwater depth ratio is different from that of the deeper jets. In the shallow jets (η = 5.3, 10.0, 15.0 & 20.0) the relative discharge increases with relative distance at a rate greater than that of the classical wall jet up to a certain section and then decreases rapidly until it eventually reaches the value of 1. Secondly, as the tailwater depth ratio increases, the maximum value of the relative discharge increases in value from 2.0 for η = 5.3 to about 3.0 for η = 20.0. Moreover, the longitudinal distance, at which this maximum relative discharge occurs, increases with increasing values of η. For the deep jets, they appear to conform better to the trend of the classical wall jets. The relative discharge increases with relative distance at the same rate as the classical wall jet and then deviates from it at a distance x/b_o is equal to about 80. Once again, the effect of the Froude number on the variation of the relative discharge with the relative distance is insignificant while the only important variable is the tailwater depth ratio η.

[5] showed that the momentum flux in the forward flow region of the wall jet is not preserved and mentioned that the wall jet could lose a substantial portion of its momentum flux as it travels downstream when the tailwater depth is relatively shallow, because of the entrainment of the re-circulating flow with opposite momentum. Fig. 10 shows the variation of the

normalized momentum flux in the forward flow of the wall jet with the normalized distance from the slot for different tailwater depth ratios and different Froude numbers. Fig. 10 confirms the distinctive difference between the behavior of the relatively shallow and the deep jets. Fig 10 shows that for the shallow jets (η = 5.3, 10.0, 15.0 & 20.0), there is a rapid increase in the momentum flux in the forward flow up to a certain section and then the momentum flux decays rapidly to less than about 20% of that at the source. Secondly, as the η increases the decay is less rapid as M/M_o reaches its minimum value at longitudinal distance of about $30b_o$ for η = 5.3 and at about $80b_o$ for η = 20.0. For the deeper jets on the other hand (η = 50.0), there is no increase in the momentum flux of the forward flow near the slot. The momentum flux is preserved up to a longitudinal distance of about $80b_o$ and then it starts to decay. Further, Fig. 10 shows the insignificance of the Froude number on the variation of the momentum flux with the relative distance.

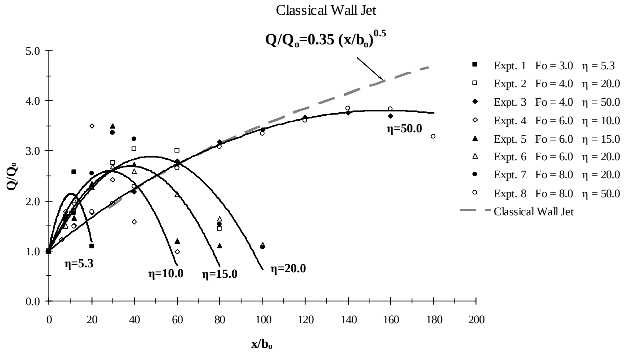

Fig. 9 Variation of wall jet discharge with distance

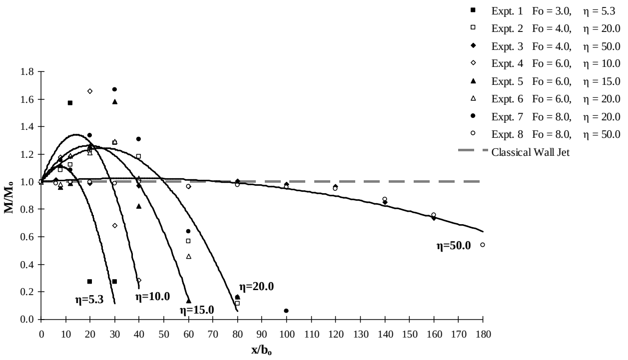

Fig. 10 Variation of wall jet momentum with distance

The difference in behavior between the shallow and deep jets regarding the variation in the momentum flux and the relative discharge of the forward flow with longitudinal distance can be attributed to the effect of the entrained flow which approaches the jet with an angle. In case of shallower jets, the surface eddy at the slot above the jet causes the entrained flow to approach the jet at a small angle to the horizontal from the backward direction. This creates a significant positive momentum component and causes the rapid increase in both the discharge and the momentum flux. In case of deeper jets the loss of the momentum flux may be attributed to the negative momentum carried by the entrained flow which approaches the jet at an angle from the forward direction.

IV. CONCLUSIONS AND RECOMENDATIONS

For turbulent wall jets in limited tailwater depth, it has been shown that there is a distinctive difference between the behavior of the jets with relatively shallower tailwater depth ratios and those with relatively deeper tailwater depth ratios. The experimental results showed that for the jets with relatively shallow tailwater depth ratios the decay of the maximum velocity is more rapid than that for the deeper jets and the variation of the normalized maximum velocity with the normalized distance is linear. Further the experimental results showed that for the shallower jets the relative discharge increases with the normalized longitudinal distance up to a maximum value then decreases rapidly to a value of 1 (at the end of the surface eddy). For the deeper jets, the relative discharge increases at a much slower rate. The distinct difference between the shallow and deep jets was more profound in the variation of the momentum flux of the forward flow with the normalized longitudinal distance. For the relatively deep tailwater depth ratios, the momentum was preserved for some distance and then started to decay at a longitudinal distance of $0.80b_o$. The study showed that for the shallower jets, the momentum flux increased rapidly to a maximum value before it started to decay and the decay occurred rapidly as well. Furthermore, the study showed that the Froude number has an insignificant effect on the decay of the maximum velocity, expansion of the jet half-width the variation of the relative discharge and the momentum flux. On the whole, the results of this study highlight the significant effect of the tailwater depth ratio on the behavior of wall jets.
It is suggested to run experiments on rough beds to discuss the effect of roughness on the behavior of plane turbulent wall jets in limited tailwater depth.

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