Sliding Mode Control of a Single Rigid Hydraulically Actuated Manipulator

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Abstract—In this paper, controlling a rigid robot manipulator driven by hydraulic system is considered. Due to changing of inertia moment of the manipulator as well as the effect of friction of hydraulic system, the dynamics of the robot arm is highly nonlinear. Therefore sliding mode control method is applied. Sliding mode control is a robust control method. It provides a symmetric approach to the problem of maintaining stability and consistent performance in the face of modeling imprecision. Numerical simulations and experiment results of the control system are presented. The results show the extremely good robustness of the proposed method.

Index Term—Hydraulic actuators, Robot control, Sliding mode control, Servosystems

I. INTRODUCTION

Hydraulic robots and machinery are widely used in the construction and mining industries as crane, excavator, robotic [5], [12], [13], [14]. They have rapid responses and high power-to-weight ratios suitable for many applications. Furthermore, the potential complexity of such controllers is becoming less and less of an implementation issue due to the inexpensive and powerful processors available today for real-time.

The control of hydraulic manipulators is more challenging that of their electrical counterparts because of the highly nonlinear hydraulic dynamics [8], [9]. Non-linear characteristics originate from the compressibility of the fluid and complex characteristic of servo valve. In addition, significant uncertain nonlinearities such as external disturbance, leakages and friction are unknown and can be not modeled accurately. Therefore the classical control methods as PI, PID. can be not applied effectively to control hydraulic manipulators as hydraulic actuators cannot accurately apply forces or torques over a significant dynamic range. It is very important to find a suitable nonlinear control method to hydraulic manipulators. In this paper, a sliding mode control is applied to control a single rigid manipulator driven by electro-hydraulic system.

Sliding mode control (SMC) or also called variable structure, is a nonlinear control method, which provides a symmetric approach to the problem of maintaining stability and consistent performance in the face of modeling imprecision and disturbances [1],[2],[3],[7]. Many papers used this method to control and showed good results with the effects of un-modeled parameters. In [2], sliding mode control is applied to control a flexible load. To enhance control performance, sliding mode control combined with fuzzy PI controller [6]. But most of them used a symmetric cylinder and so the dynamic of system became simpler as well as controlling. When an asymmetric cylinder is applied, dynamics of the system becomes more complex and using sliding mode control with only a feedback of their position is more difficult. So a new approach of sliding mode control is used. In this way, both errors of position and load pressure is feed backed to design a controller [2], [3]. This method so far just applied for applications of straight motions, not for manipulators.

In this paper, the method originates from [2], [3], and is applied to control a single rigid manipulator. Angle of the manipulator is tracked by following a desired reference angle. Here, pressure between two chambers of cylinder is combined to create a load pressure error of load pressure. Errors of load pressure and angle position are applied to design a controller. Numerical simulation is presented and gives a good tracking.

This paper is organized as follow: section II introduces dynamic formulation and problem statement, section III discusses controller and estimator designs. Section IV shows the simulation results, section V presents the experiment results and section VI concludes the paper.

II. FORMULATION AND PROBLEM STATEMENT

This paper focuses on the single rigid arm control driven by hydraulic system. The coordinate systems, joint angles and physical parameters of the system are defined as in Fig. 1.
The kinetic (T) and potential (U) energy of the arm can be determined as [10], [11].

\[
T = \frac{1}{2} \left( ML^2 + \frac{mL^2}{3} \right) \dot{\theta}^2 = \frac{1}{2} \left( M + \frac{m}{3} \right) L^2 \dot{\theta}^2
\]

(1)

\[U = g \left( M + \frac{m}{2} \right) L \sin \theta\]

Applying the Lagrange’s method, the equation of motion are obtained as

\[
d\left( \frac{\partial T}{\partial \dot{\theta}} \right) - \frac{\partial T}{\partial \theta} + \frac{\partial U}{\partial \theta} = R_o
\]

(2)

where \( R_o \) is the generalized force corresponding to the generalized coordinate \( \theta \)

\[R_o = \tau_f - T_f(\dot{\theta})\]

(3)

where \( T_f(\dot{\theta}) \) is the torque due to friction, is expressed into [3].

\[T_f(\dot{\theta}) = b_1 \text{sgn}(\dot{\theta}) + b_2 \dot{\theta}\]

(4)

with \( b_1 \) and \( b_2 \) are constant positive coefficients. \( \tau_f \) is the torque of the hydraulic actuator force \( F \) on the arm.

\[\tau_f = F \frac{\partial y}{\partial \theta}\]

(5)

with \( y \) is a new variable and is calculated from Fig.1

\[y = \sqrt{l_1^2 + l_0^2 - 2ll_0 \cos(\theta + \alpha)} - x_0\]

(6)

where, \( x_0 \) is length of the cylinder.

Substituting (1), (3)-(6) into (2), gives the equation of the single rigid arm.

\[
\left( M + \frac{m}{3} \right) L^2 \ddot{\theta} + \left( M + \frac{m}{2} \right) g L \cos \theta + T_f(\dot{\theta}) = \left( P_1A_1 - P_2A_2 \right) \frac{\partial x}{\partial \theta}
\]

(7)

By putting,

\[
H_1 = \left( M + \frac{m}{3} \right) L^2
\]

\[
H_2 = \left( M + \frac{1}{2} m \right) g L
\]

(8)

and noting that the joint displacement, \( \theta \), and the piston displacement, \( x \), are related by a geometrical configuration.

Within the vicinity of certain angle \( \hat{\theta} \), the following relation holds

\[dx = \text{sgn}(\dot{\theta}) d\theta\]

(9)

where,

\[
\text{sgn}(\dot{\theta}) = \frac{2ll_0 \sin \hat{\theta}}{\sqrt{l_1^2 + l_0^2 - 2ll_0 \cos(\alpha + \hat{\theta})}}
\]

Since \( T_f(\dot{\theta}) = T_1 \text{sgn}(\dot{\theta}) + T_2 \dot{\theta} \), then (8) becomes

\[
H_1 \ddot{\theta} + H_2 \cos \theta + T_1 \text{sgn}(\dot{\theta}) + T_2 \dot{\theta} = F \text{sgn}(\dot{\theta})
\]

(10)

where \( T_1 \) and \( T_2 \) are Coulomb and viscous friction coefficients respectively.

The schematic of the hydraulic servo system driving the single rigid manipulator is showed in Fig. 2. Here \( P_s \) and \( P_t \) are the supply pressure and the tank reference pressure.

\[
H_1 \ddot{\theta} + H_2 \cos \theta = \frac{2ll_0 \sin \hat{\theta}}{\sqrt{l_1^2 + l_0^2 - 2ll_0 \cos(\alpha + \hat{\theta})}}
\]

(11)

Assuming that the time constant of the servo valve motor/flapper is much smaller than those of the mechanical parts, we can consider the spool displacement \( x_v \) is proportional with the current input \( u \) of the servo valve.

\[x_v = K_s u\]

(12)

where, \( K_s \) is a proportional gain of the servo-valve.

The amount of fluid flow to the head-side \( Q_1 \) and from the rod-side \( Q_2 \) of the cylinder is a function of both the valve spool position and cylinder pressures. The relationship can be expressed in the following form [8]:

\[Q_1 = k_v g_1(P_1, \text{sgn}(x_v))x_v\]

\[Q_2 = k_v g_2(P_2, \text{sgn}(x_v))x_v\]

(13)

where \( k_v \) is the flow gain coefficient of the servo valve, \( g_1 \) and \( g_2 \) are functions of \( x_v \) and \( P_1, P_2 \) that given by (13).
\[
g_1(P_s, \text{sgn}(x_s)) = \begin{cases} 
\sqrt{P_s - P_1}, & x_s \geq 0 \\
\sqrt{P_1 - P_s}, & x_s < 0 
\end{cases}
\]
(13)

\[
g_2(P_s, \text{sgn}(x_s)) = \begin{cases} 
\sqrt{P_s - P_2}, & x_s \geq 0 \\
\sqrt{P_2 - P_s}, & x_s < 0 
\end{cases}
\]
(14)

Applying the flowing continuity equations to the two sides of the cylinder and neglecting any external leakage [8]:

\[
Q_1 = A_1 \frac{dx}{dt} + \frac{V_1}{\beta_c} \dot{P}_1
\]
(15)

\[
Q_2 = A_2 \frac{dx}{dt} - \frac{V_2}{\beta_c} \dot{P}_2
\]
(16)

where \(\beta_c\) is the effective bulk modulus, \(V_1\) and \(V_2\) are the volume of two chambers of the cylinder respectively, calculated as

\[
V_1 = V_o + A_1x
\]
\[
V_2 = V_o + A_2(L_o - x)
\]
(17)

where \(V_o\) and \(L_o\) are the dead volume and the stroke of the cylinder, respectively.

By solving for \(\dot{P}_1\) and \(\dot{P}_2\) in (14), the dynamics of the cylinder and its valve can be written as

\[
\dot{P}_1 = \frac{\beta_c}{V_1} \left( Q_1 - A_1 \frac{dx}{dt} \right)
\]
(18)

\[
\dot{P}_2 = \frac{\beta_c}{V_2} \left( A_2 \frac{dx}{dt} - Q_2 \right)
\]
(19)

The force \(F\) applied by the cylinder to the arm is simply

\[
F = P_1A_1 - P_2A_2
\]
(20)

where \(P_1\) and \(P_2\) are the head and rod end pressure of the cylinder, \(A_1\) and \(A_2\) are the head and rod areas of the cylinder respectively.

Substituting \(F\) into (10) the dynamic equation of the entire system is obtained as

\[
H_1\ddot{\theta} + H_2 \cos \theta + T_1 \text{sgn} (\dot{\theta}) + T_2 \dot{\theta} = (P_1A_1 - P_2A_2) \text{sgn} (\dot{\theta})
\]
(21)

For a small variation about the reference point, dynamic (18) can be linearized as follow,

\[
H_1\ddot{\theta} + T_2 \dot{\theta} \approx (P_1A_1 - P_2A_2) \text{sgn} (\dot{\theta})
\]
(22)

Linearizing the valve dynamics (12) and rewriting the results in \(s\)-domain gives, [17]

\[
Q_1 = K_u U - K_p P_1
\]
\[
Q_2 = K_u U + K_p P_2
\]
(23)

\[
K_p = \frac{k_u x}{\sqrt{2(P_s - P_L)}}
\]
(24)

Transforming the flowing continuity (16) into \(p\)-domain gives

\[
Q_1 = CP_p p + A_1xp
\]
\[
Q_2 = -CP_p p + A_2xp
\]
(25)

In (24), it is assumed that \(C = \frac{V_1}{\beta_c} = \frac{V_2}{\beta_c}\)

Combining (20) and (24) yields

\[
P_1 = \frac{K_u U - A_1 p \text{sgn} (\dot{\theta})}{K_p + Cp}
\]
\[
P_2 = -\frac{K_u U + A_2 p \text{sgn} (\dot{\theta})}{K_p + Cp}
\]
(26)

Substituting \(P_1\) and \(P_2\) into (19) and taking Laplace transform yields

\[
\left( H_1 p^2 + T_2 p \right) \ddot{\theta} = \frac{K_u (A_1 + A_2) \text{sgn} (\dot{\theta}) U - (A_1^2 + A_2^2) \text{sgn} (\dot{\theta})}{K_p + Cp} \frac{\text{sgn} (\dot{\theta})}{K_p + Cp}
\]
(27)

Equation (27) can be written as

\[
\theta (p) = \frac{b_3}{p(p^2 + a_1p + a_2)}
\]
(28)

where

\[
a_1 = \frac{H_1 K_p + T_2 C}{H_1 C}
\]
\[
a_2 = \frac{\text{sgn} (\dot{\theta})(A_1^2 + A_2^2) + T_2 K_p}{H_1 C}
\]
\[
b_3 = \frac{K_u (A_1 + A_2) \text{sgn} (\dot{\theta})}{H_1 C}
\]
(29)

Transfer function (28) can be expressed in state space form as:

\[
x = Ax + Bu
\]
\[
y = \theta
\]
(30)

where

\[
x = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix}, \ A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \ B = \begin{bmatrix} 0 \\ b_3 \end{bmatrix}
\]
(31)

The system dynamics characteristics are inherently highly
nonlinear and may vary due to load variations and friction torque. Equation (29) is therefore can be rewritten as
\[ \dot{x} = (A_0 + \Delta A)x + (B_0 + \Delta B)u + \nu \] 
with
\[ A_0 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -a_1 & -a_2 \end{bmatrix}, \quad B_0 = \begin{bmatrix} 0 \\ 0 \\ b \end{bmatrix}, \quad \nu = \begin{bmatrix} 0 \\ 0 \\ v \end{bmatrix} \] 
where \( A_0 \) and \( B_0 \) are the nominal system matrices. \( \Delta A \) and \( \Delta B \) represent the parametric variations and/or uncertainties. \( \nu \) is the vector of disturbances and unmodelled dynamics.

### III. CONTROLLER AND ESTIMATOR DESIGN

#### A. CONTROLLER DESIGN

Recall (30). The following assumptions are taken:

1. \( \Delta A \) and \( \Delta B \) are continuous matrix functions of a vector of uncertain parameters \( \rho \).
   \[ \Delta A = \Delta A(x, \rho, t) \]
   \[ \Delta B = \Delta B(x, \rho, t) \] 

2. There exist matrices and two scalars and such that the following matching conditions are satisfied:
   \[ \Delta A = B \tilde{A} \quad \text{and} \quad \max \{\tilde{\alpha}_i\} \leq \alpha_i, \quad i = 1, 2 \]
   \[ \Delta B = B \tilde{b} \quad \text{and} \quad \tilde{\beta} \leq \beta < 1 \]
   \[ \nu = B \tilde{\nu} \quad \text{and} \quad \tilde{\nu} \leq \nu \] 

The control objective is to design a sliding mode controller that provides robust position control performance in the presence of uncertainties (33). The control system block diagram of the system is shown as Fig. 3.

![Block diagram of the control system](image)

The state error of the system is defined as
\[ e = x - x_d \] 
where, \( x_d \) is the desired input state vector as follows
\[ x_d = \begin{bmatrix} \theta_d \\ \omega_d \\ \dot{\omega}_d \end{bmatrix} \] 
and state equation of the system at steady state meets as
\[ \dot{x}_{sd} = A_0 x_{sd} + B_0 u_d \] 
where, \( u_d \) is the control law in the steady state.

By combining (30), (34), and (36), we obtain an error state equation as
\[ e = A_0 e + B_0 (u - u_d) + \Delta A x + \Delta B u + \nu \] 
Therefore, the sliding mode control can be applied with this system by defining the switching function of the form as
\[ s = C^T e \] 
where
\[ C = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \] 
The values of \( c_1 \) and \( c_2 \) are determined by using the pole assignment method. If the poles of the system dynamics in a sliding mode condition are assigned to real \( s_{01} \) and \( s_{02} \) on the \( s \)-plane, then
\[ c_1 = s_{01} s_{02} \]
\[ c_2 = -(s_{01} + s_{02}) \] 
The control input is composed of a linear and a nonlinear control terms. A linear control is the equivalent control, \( u_{eq} \), and is calculated by letting \( \Delta A = 0, \Delta B = 0, \) \( v = 0 \) and by using the condition \( \dot{s} = 0 \). From (37) and (38),
\[ u_{eq} + u_d = -(C^T b_0) i C^T A_0 e \] 
, or
\[ u_{eq} = -K_{eq} e + u_d \] 
where
\[ K_{eq} = -(C^T b_0) i C^T A_0 \] 
The nonlinear control or discontinuous term \( u_{nl} \) can simply be constructed by a signum function with a constant magnitude
\[ u_{nl} = -k \text{sgn}(s) \] 
Thus, the control input is then expressed as
\[ u = u_{eq} + u_{nl} \] 
By choosing the Lyapunov’s function be
\[ V = \frac{1}{2} s^2 \] 
and the derivative of the Lyapunov’s function is given as
\[ \dot{V} = ss \dot{s} \] 
The discontinuous gain \( k \) is determined such that \( \dot{V} < 0 \). Indeed, by taking (38), (42), (44), and (45) into consideration, (47) becomes
\[ \dot{V} = ss \dot{s} \] 
then (48) becomes
\[ \dot{s} + \tilde{b} \dot{k}_{eq} \frac{e + \tilde{b} u_d}{1 + \tilde{b} k_{eq}} \leq 0 \] 

Since
\[ C^T B_0 = b_{eq} > 0 \] 
then (48) becomes
\[ \tilde{\nu} + \tilde{b} \dot{k}_{eq} \frac{e + \tilde{b} u_d}{1 + \tilde{b} k_{eq}} \leq 0 \] 
Thus, \( k \) is determined from the condition in (50) as
\[ k > \max \left( \frac{\dot{\tilde{\nu}} + \tilde{b} u_d + \tilde{b} \dot{k}_{eq} e}{1 + \tilde{b} k_{eq}} \right) \]
or

\[ k > \frac{\nu + \beta u_d}{1 - \beta} + \sum_{i=1}^{3} \left( \alpha_i |x_i| + \beta_i |K_{eq, i} e_i| \right) \]  

(52)

In practice, discontinuity in the switching control (44) usually results in chattering which may excite undesirable high-frequency or unmodelled dynamics. Therefore, (44) is often replaced by a smoothing function such as

\[ u_w = k \frac{s}{|s| + \delta} \text{ for } \delta > 0 \]  

(53)

The value of \( \delta \) is obtained through computer simulations.

B. OBSERVER DESIGN

In SMC controller above, all states are used to implement traditional SMC controller. They are angular position, velocity and acceleration. The position is measured by a sensor, an encoder in this case. However, sensors to detect the velocity and acceleration are usually not available on electro-hydraulic servo systems, and therefore these state variables are approximately obtained by differentiating the position signal or are estimated by an observer.

When designing a controller, plant dynamics is usually described using a lower order model. This lower order approximation can sometimes cause problems resulting from unmodelled high-frequency dynamics. When implementing a sliding mode control using actual variables of a plant, chattering problems will occur. On other hand, when using the state variables estimated by an observer to calculate the switching function, chattering can be suppressed because the observer is free of unmodelled dynamics. Therefore, in this investigation, an observer is used to obtain the estimates of the velocity and acceleration as illustrated in Fig. 4.

\[
\begin{align*}
\dot{x}_d &= A_0 x + B_0 u \\
y &= \theta
\end{align*}
\]

(54)

The observer is constructed in discrete form, by discretizing (54) and using the zero-order hold equivalence method

\[
x_e(k) = \Phi x(k - 1) + \Gamma u(k - 1)
\]

(55)

where

\[
\Phi = \exp(A_0), \quad \Gamma = \int_{0}^{T} \exp(A_0 \eta) d\eta B_0
\]

(56)

The current observer is constructed using current output as shown in Fig. 5 and its equation is given as

\[
\begin{align*}
\tilde{x}(k) &= \Phi \tilde{x}(k - 1) + \Gamma u(k - 1) \\
\tilde{y}(k) &= \tilde{\theta}(k) \\
\dot{\tilde{x}}(k) &= \tilde{x}(k) + L_c [\theta(k) - H \tilde{y}(k)]
\end{align*}
\]

(57)

where, \( \tilde{x}(k) \) is the predicted estimate based on the model prediction from the previous time estimate. \( \tilde{x}(k) \) is the current estimate based on the current measurement \( \theta(k) \). The observer gain vector \( L_c \) is determined by pole assignment.

IV. SIMULATIONS AND RESULTS

In order to demonstrate the effectiveness of the proposed robust control method, numerical simulations are performed. The simulations were conducted with the MATLAB/SIMULINK. The sliding mode controller is simulated by the fourth-order Runge-Kutta method with the time interval, \( T = 0.004s \) and the bounds of uncertainties in the condition (33) can be determined as: \( \alpha_i = 0.3|\alpha_i|, i = 1, 2, 3, \beta = 0.3|\beta_e| \) and \( \nu = 0.5 \). The desired poles of the system dynamics in a sliding mode condition are chosen as, \( s_{01} = -72 \) and \( s_{02} = -8 \). Thus, the values of \( c_1 \) and \( c_2 \) are 600 and 80, respectively. The desired poles of the observer are chosen at 0.6 and the controller tuning parameters are selected as \( k = 15 \) and \( \delta = 500 \).
Fig. 7. Tracking error for rectangular input (a) for $M = 0$ kg and (b) for $M = 130$ kg

Fig. 8. Control input current for rectangular input (a) for $M = 0$ kg and (b) for $M = 130$ kg

Fig. 9. Desired sine input trajectory

Fig. 10. Tracking error for sinusoidal input (a) for $M = 0$ kg and (b) for $M = 130$ kg

Fig. 11. Control input for sinusoidal input (a) for $M = 0$ kg and (b) for $M = 130$ kg

Fig. 12. Desired triangular input trajectory
V. EXPERIMENT RESULTS

Figure 15 shows the experiment setup that consists of a single arm manipulator with a payload driven by a hydraulic cylinder. Hydraulic flow to the cylinder is controlled by a servovalve. To demonstrate the robustness of the proposed controller to parameter uncertainties, the payload is varied from $M = 0$ kg to $M = 130$ kg. The proposed controller is implemented at 5.3 msec. sampling time. The experimental parameters and controller parameters are mainly the same as those used in the simulations. Experiments are carried out with the same inputs which were used in the simulation and therefore the experimental results could be compared to the simulation results.
Fig. 18. Tracking error to sinusoidal input for (a) $M = 0$ kg, (b) $M = 130$ kg

Fig. 19. Control input for sinusoidal input (a) for $M = 0$ kg and (b) for $M = 130$ kg

Fig. 20. Tracking error for triangular input for (a) $M = 0$ kg, (b) $M = 130$ kg

Fig. 21. Control input current for triangular input (a) for $M = 0$ kg and (b) for $M = 130$ kg

Fig. 16, 18, and 20 show the position tracking errors, and control laws respectively. For the rectangular input, there is small overshoot in transient state and no error for steady state. The controller however took about 0.8s ($M = 0$ kg) to achieve 10° from initial position of 0° and 1.0s for $M = 130$ kg. It is slower in comparison with the response in simulation. This is because of the flow limitation of the hydraulic pump required to supply the cylinder. For the sinusoidal input, $x_{id} = 10 \sin(0.2\pi t)$, the maximum tracking error in steady state is less than 0.5° ($M = 0$ kg) and 0.6° ($M = 130$ kg). For the triangular wave input the maximum tracking error in steady state is less than 0.5° ($M = 0$ kg) and 0.6° ($M = 130$ kg). Fig. 17, 19, and 21 show that there are only small chattering appear in the control law. This demonstrates that the proposed sliding mode controller could eliminate the chattering beside its robust control performances.

VI. CONCLUSION

A sliding mode control has been proposed and applied to a rigid manipulator. The combination of angular position error and the load pressure error can be asymptotically tracked even
when the system is subject to parameter uncertainties. Numerical simulation results have shown good performances of tracking.

Experimental has been carried out to investigate the effectiveness of the proposed control method. Experimental results showed similar performance to these simulation results. Rapid response and small tracking errors were close to the simulation results. The proposed controller has high robustness to load variation. There were chatters in control laws in those cases. However, these chatters were very small and acceptable in operating condition of the system.

The main contribution of this paper is to find a new solution to reduce the chattering problem in sliding mode control scheme to the control of a single-hydraulically actuated manipulator. Instead of using common boundary layer approach, an observer to estimate variable states of the controller: velocity and acceleration is applied to reduce chattering problem. The experiment results show that the proposed controller is able achieve good tracking results without chattering problem.

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