Numerical study of MHD free convection flow and mass transfer over a stretching sheet considering Dufour & Soret effects in the presence of Magnetic Field

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Abstract

In the present approach, a two-dimensional steady MHD free convection flow and mass transfer over a stretching sheet has been analyzed numerically including the Dufour and Soret effects with a magnetic field. The governing differential equations of the problem have been transformed into a system of non-dimensional differential equations, which are then solved numerically using a sixth-order Runge-Kutta integration scheme with Nachtsheim-Swigert shooting method. The dimensionless velocity, temperature and concentration profiles are displayed graphically showing the effects for the different values of the involved parameters of the problem. Moreover, the effects of *So* and *Du* on the local skin-friction coefficient (*Cf*), local Nusselt number (*Nu*) and local Sherwood number (*Sh*) are also shown in tabular form. The investigated results showed that the flow field is notably influenced by the considering parameters.

Keywords: MHD, Convection, Flow and mass transfer, Dufour effect, Soret effect, Magnetic field.

NOMENCLATURE:

A_1, A_2 are proportionality constants
b Stretching rate
B_0 Magnetic field intensity
C Concentration
C_s Concentration susceptibility
C_p Specific heat at constant pressure
C_∞ Concentration in free stream
Dm Mass diffusivity
Du Dufour number
fw Dimensionless suction velocity
Fs Local Forchheimer number
g Acceleration due to gravity
Gm Local modified Grashof number
Gr Local Grashof number
K_T Thermal diffusion ratio
k Thermal conductivity
M Magnetic field parameter
Nu Nusselt number
Pr Prandtl number
r heat flux exponent parameter
Re Local Reynolds number
Sc Schmidt number
Sh Sherwood number
So Soret number
T Fluid temperature
T_∞ Fluid temperature in the free stream
T_m Mean fluid temperature
α Thermal diffusivity
β Coefficient of thermal expansion
β^* Coefficient of concentration expansion
σ Electrical conductivity
ρ Density of the fluid
ν Kinematic viscosity
θ Dimensionless temperature
φ Dimensionless concentration
w Condition at wall
∞ Condition at infinity
u, v Darcian velocities in the x and y-direction respectively
x, y Cartesian coordinates along the plate and normal to it, respectively

1. Introduction

The study of Magnetohydrodynamic (MHD) flows have stimulated extensive attention due to its significant applications in three different subject areas, such as astrophysical, geophysical and engineering problems.Free convection in electrically conducting fluids through an external magnetic field has been a subject of considerable research interest of a large number of scholars for a long time due to its diverse applications in the fields such as nuclear reactors, geothermal engineering, liquid metals and plasma flows, among others. Fluid flow control under magnetic forces is also applicable in magnetohydrodynamic generators and a host of magnetic devices used in industries. Steady and transient free convection of an electrically conducting fluid from a vertical plate in the presence of magnetic field was studied by Gupta [1]. Lykoudis [2] investigated natural convection of an electrically conducting fluid with a magnetic field. Takhar et al. [3] computed flow and mass transfer on a stretching sheet under the consideration of magnetic field and chemically reactive species. They focused that the energy flux can be produced by both of the temperature gradient and concentration gradient. The energy flux caused by concentration gradient is called Dufour effect and the same by temperature gradient is called the Soret effect. These effects have a vital role in the high temperature and high concentration gradient. The significant Soret effects in convective transport in clear fluids has been found in the work of Bergaman and Srinivasan [4] and Zimmerman et al. [5]. The effect of magnetic field on heat and mass transfer from vertical surfaces in porous media considering Soret and Dufour effects have been performed by Postelnicu [6]. Alam et al. [7] analyzed the Dufour and Soret effects on steady MHD combined free forced convective and mass transfer flow past a semi-infinite vertical plate. Jha et al. [8] included Soret effects free convection and mass transfer flow in the stokes problem for an infinite vertical plate. An analysis of the steady two-dimensional flow of an incompressible viscous and electrically conducting fluid over a non-linearly semi-infinite stretching sheet in the presence of a chemical reaction and under the influence of a magnetic field have been carried out by Raptis and Perdikis [10]. Acharya et al. [11] studied heat and mass transfer over an accelerating surface with heat source in the presence of suction and blowing. Anghel et al. [13] investigated the Dufour and Soret effects on free convection boundary layer over a vertical surface embedded in a porous medium. Mohammadein et al. [14] found the result of heat transfer in a micropolar fluid over a stretching sheet with viscous dissipation and internal heat generation. Helmy [15] presented the effects of the magnetic field on a non-Newtonian conducting fluid past a stretching plate. The goal of the present work is to investigate the effects of Dufour number and Soret number in the presence of magnetic field for MHD free convection flow and mass transfer along a stretching sheet.

2. Governing equations of the flow

In the present problem, it can be considered that the flow is steady, two-dimensional, laminar MHD free convective, viscous and incompressible along a linearly stretching semi-infinite sheet. The surface is supposed to be permeable which moves with velocity $u_w(x) = bx$ and $v_w(x)$ represents the permeability of the porous surface. Fluid suction is imposed at the stretching surface. The *x*-axis runs along the stretching surface in the direction of motion with the slot as the origin and the *y*-axis is measured normally from the sheet to the fluid. The applied magnetic field is primarily in the *y*-direction and varies in strength as a function of *x* and is defined as $\boldsymbol{B} = (0, B(x))$. Moreover, the electrical conductivity σ is assumed to have the form as $\sigma = \sigma_0 u$, where σ_0 is a constant.

According to the usual Boussinesq and boundary-layer approximation, the governing equations for this problem can be written as follows:

Continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

Momentum equation

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)+g\beta^*(C-C_\infty)-\frac{\sigma(B(x))^2}{\rho}u, \tag{2}$$

Energy equation

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{D_m k_T}{c_s c_p}\frac{\partial^2 C}{\partial y^2}, \tag{3}$$

Concentration equation

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2}+\frac{D_m k_T}{T_m}\frac{\partial^2 T}{\partial y^2}, \tag{4}$$

It is appropriate to suppose that the applied magnetic field strength $B(x)$ has the form [Helmy (1994)]: $B(x)=\dfrac{B_0}{\sqrt{x}}$, B_0 is constant. (5)

Using equation (5), the fourth term in equation (2) can be rewritten as:

$$\frac{\sigma(B(x))^2 u}{\rho}=\frac{\sigma_0 B_0^2 u^2}{\rho x}, \text{ where } \sigma=\sigma_0 u, \tag{6}$$

Therefore by means of equation (6), equation (2) reduces to

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)+g\beta^*(C-C_\infty)-\frac{\sigma_0 B_0^2}{\rho x}u^2, \tag{7}$$

The suitable boundary conditions are given by:

$$u=u_w(x)=bx,\ v=\pm\, v_w(x),\ -k\frac{\partial T}{\partial y}=q_w=A_1x^r,\ -D_m\frac{\partial C}{\partial y}=M_w=A_2x^r \text{ at } y=0, \tag{8a}$$

$$u=0,\quad T=T_\infty,\quad C=C_\infty \quad \text{at} \quad y\to\infty\,. \tag{8b}$$

By mean of the following similarity variables [Acharya et al. (1999)];

$$u=\frac{\partial\psi}{\partial y},\ v=-\frac{\partial\psi}{\partial x},\ \psi=(\upsilon b)^{1/2}xf(\eta),\ \eta=(b/\upsilon)^{1/2}y,$$

$$T-T_\infty=\frac{A_1x^r}{k}(\upsilon/b)^{1/2}\theta(\eta),\ C-C_\infty=\frac{A_2x^r}{D_m}(\upsilon/b)^{1/2}\varphi(\eta).$$

Equations (7), (3) and (4) be converted into the equations as below:

$$f'''+ff''-(f')^2+g_s\theta+g_c\varphi-M(f')^2=0, \tag{9}$$

$$\theta''-r\Pr f'\theta+\Pr f\theta'+\Pr Df\phi''=0, \tag{10}$$

$$\varphi''-rScf'\varphi+Scf\varphi'+ScSr\theta''=0\,. \tag{11}$$

The transformed boundary conditions are given by

$$f=f_w,\ f'=1,\theta'=-1,\phi'=-1 \text{ at } \eta=0, \tag{12a}$$

$$f'=0,\ \theta=0,\ \phi=0 \quad \text{at} \quad \eta\to\infty, \tag{12b}$$

where the dimensionless wall mass transfer coefficient defined as $f_w=-v_w/(b\upsilon)^{1/2}$, whose positive & negative value indicates wall suction and wall injection respectively.

The dimensionless parameters introduced in the above equations are defined as follows:

$M = \frac{\sigma B_0^2 x}{\rho u_w(x)}$ is the local magnetic field parameter, $Gr = \frac{g\beta q_w x^4}{k\upsilon^2}$ is the local Grashof number, $Gm = \frac{g\beta^* M_w x^4}{D_m \upsilon^2}$ is the local modified Grashof number, $\mathrm{Re}_x = \frac{u_w(x)x}{\upsilon}$ is the local Reynolds number, $g_s = \frac{Gr}{\mathrm{Re}_x^{5/2}}$ is the temperature buoyancy parameter, $g_c = \frac{Gm}{\mathrm{Re}_x^{5/2}}$ is the mass buoyancy parameter, $Du = \frac{D_m M_w k_T}{\upsilon c_s c_p q_w}$ is the Dufour number, $So = \frac{D_m q_w k_T}{k M_w T_m}$ is the Soret number, $\Pr = \frac{\upsilon \rho C_p}{k}$ is the Prandtl number, $Sc = \frac{\upsilon}{D_m}$ is the Schmidt number.

Nachtsheim-Swigert (1965) shooting iteration technique together with Runge-Kutta sixth-order integration procedure have been used for solving the system of equations (9), (10), (11) under the boundary conditions (12). $\Delta\eta = 0.01$ was selected as the step size that satisfied a convergence criterion of 10^{-6} for the calculations of parameters in different phases. From the process of numerical computation, the skin-friction coefficient, the local Nusselt number and the local Sherwood number are proportional to $f''(0)$, $\frac{1}{\theta(0)}$, $\frac{1}{\varphi(0)}$ respectively.

3. Results and discussion

Numerical computations have been performed for different values of M, So, Du and for fixed values of $Pr = 0.71$ (air), $Sc = 0.22$ (hydrogen), $g_s = 12$; $g_c = 6$ (due to free convection problem). The values of the Soret number So and Dufour number Du are assumed in such a way that their product is constant provided that the mean temperature T_m is kept constant as well. With the above-mentioned flow parameters, the results are displayed in Figs. 1–6, for the velocity, temperature and concentration profiles. In addition, the local skin-friction coefficient, the local Nusselt number and the local Sherwood number are also shown in table-2 for different values of *So* and *Du*.

Figs. 1, 2, & 3 present the behavior of the velocity f', temperature θ and concentration φ profiles for various values of the magnetic field parameter M. The presence of a magnetic field normal to the flow in an electrically conducting fluid produces a body force against the flow. This resistive force tends to slow down the motion of the fluid in the boundary layer and thus it is seen that in Fig.1 the fluid velocity decreases with the increase of the magnetic field parameter. Fig.2 appears in increasing the flow temperature as the magnetic field parameter increases. This indicates that the fluid is heated by the applied magnetic field and consequently reduces the heat transfer from the wall. Finally, the concentration profiles is increased with increasing the values of the magnetic field parameter, and so the concentration boundary layer enhances.

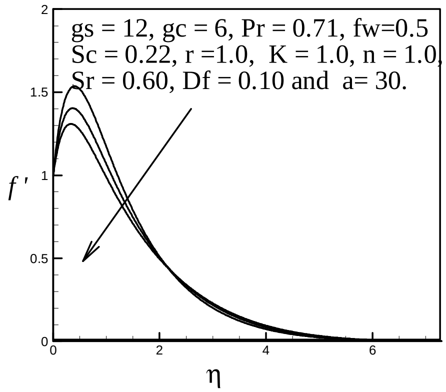

Fig.1. Velocity profiles for different values of Magnetic field parameter

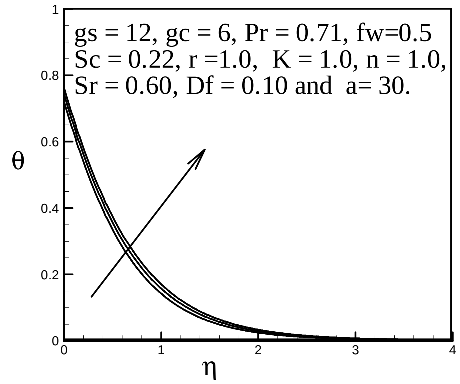

Fig.2.Temprature profiles for different values of Magnetic field parameter

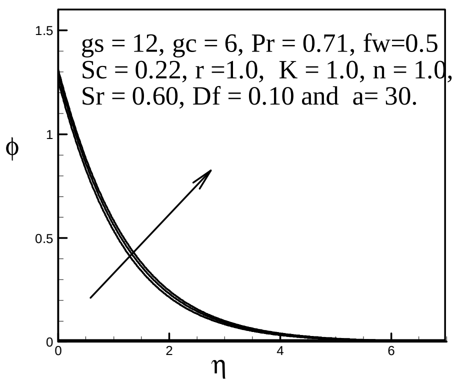

Fig.3. Concentration profiles for different values of Magnetic field parameter

The effects of Soret number *So* and Dufour number *Du* on the velocity field, temperature field and concentration field are displayed in Figs.4, 5, & 6 and their numerical values are listed in Table-1 (quantitatively, when $\eta = 0.5$). This results can be expressed shortly in the following ways;

In step-1 (*So* decreases from 1.2 to 0.8 or *Du* increases from 0.050 to 0.075), there is 1.49% and 6.12% decreases in velocity value and concentration value respectively whereas 2.86% increase in temperature value.

In step-2 (*So* decreases from 0.8 to 0.4 or *Du* increases from 0.075 to 0.150), there is 0.87% and 6.87% decreases in velocity value and concentration value respectively whereas 5.85% increase in temperature value.

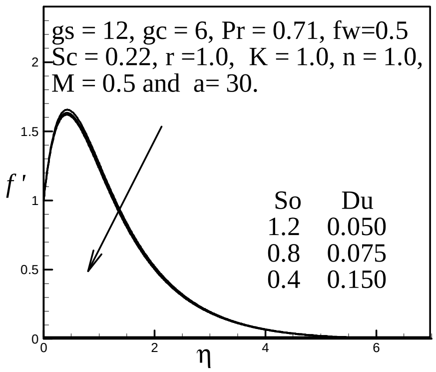

Fig.4. Velocity profiles for different values of *So* & *Du*

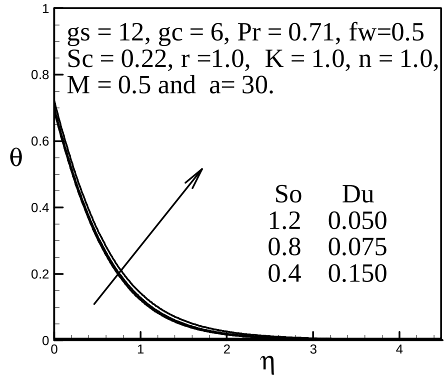

Fig.5. Temperature profiles for different values of *So* & *Du*

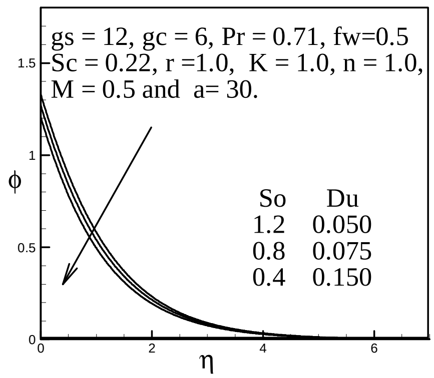

Fig.6. Concentration profiles for different values of *So* & *Du*

Table-1: Comparison of the values of velocity, temperature and concentration for different values of *So* & *Du* with η = 0.5 and for g_s = 12, g_c = 6, *Pr* =0.71, *Sc* = 0.22, *r* =1.0, f_w = 0.50, *K*=1.0, *M* = 0.50, *n* = 1.0.

Step	η	*(So, Du)*	f'	Decrease	θ	Increase	φ	Decrease
1	0.5	(1.2, 0.050)	1.6452	1.49%	0.3072	2.86%	0.8940	6.12%
		(0.8, 0.075)	1.6207		0.3160		0.8393	
2	0.5	(0.8, 0.075)	1.6207	0.87%	0.3160	5.85%	0.8393	6.87%
		(0.4, 0.150)	1.6066		0.3345		0.7816	

Moreover, Table-2 focused the shared effects of Soret number and Dufour number on the local skin-friction coefficient, the local Nusselt number and the local Sherwood number.

Table-2: Numerical values of *Cf* , *Nu* and *Sh* for different values of *So* & *Du* with $g_s = 12$, $g_c = 6$, $Pr = 0.71$, $Sc = 0.22$, $r = 1.0$, $f_w = 0.50$, $K = 1.0$, $M = 0.50$, $n = 1.0$.

(So, Du)	*Cf*	*Nu*	*Sh*
(1.2 , 0.050)	3.7967	1.4461	0.7518
(1.0, 0.060)	3.7451	1.4374	0.7692
(0.8, 0.075)	3.6975	1.4263	0.7877
(0.6, 0.100)	3.6586	1.4110	0.8074
(0.4, 0.150)	3.6417	1.3858	0.8292

4. Conclusions

The problem of steady-state, incompressible, viscous, laminar and MHD free convection flow along a stretching sheet in the presence of magnetic field have been investigated. The important conclusions have been drawn as follows:

_ The velocity profiles decrease whereas temperature and concentration profiles increase by rising magnetic field parameter.
_ For the combined effects of Dufour and Soret numbers (*Du* inreases and thus *So* decreases), the temperature profiles increase while velocity and concentration profiles decrease.
_ The local skin-friction coefficient and the local Nusselt number decreases with increasing the Dufour number (decreasing Soret number) while the local Sherwood number increases.

5. References

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