An Integer Linear Programming Model and Heuristics Scheduling EDD for Hardboards Cutting Problem in a Modular Production System

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***Abstract*—In this paper, a two-phase algorithm was developed for the cutting sequencing problems in a modular manufacturing system. This system has a level of flexibility that depends on the cutting phase of raw material. This paper focuses primarily on the work station. In the first phase of the algorithm, an integer linear programming model is used to determine the number of hardboards that will be cut. The model was tested with two different objective functions which are to minimize waste and to minimize the number of hardboards used. In the second phase, a heuristic scheduling was developed to determine the pattern of cutting, by considering the due date and the number of customer demand. This algorithm is further implemented in a furniture manufacturer that operates using the make-to-order basis.**

***Index Terms*—EDD scheduling, integer linear programming, manufacturing system, sequential production.**

I. INTRODUCTION

Currently, a rapid industrial development leads to high competition. To be able to survive in the competition and the development of this industry, the response of companies to meet consumer demand for products that often change at any time become one of the important goals for each manufacturing company who wants to achieve success. One of them is the timber industry must always deal with consumer demand for wood products size variations produced.

Wood processing industry usually considers the phase of cutting wood in minimizing the residual waste or congestion in the work station. Much research has been done to solve the problems. This can be seen from the formulation of the first model proposed by Haessler and Sweeney (1991), Yuen and Richardson (1995), Foronda and Carino (1991), Yanasse (1991), Carnieri et al. (1994), Morabito and Garcia (1998) and Parwadi (2009)

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Haessler and Sweeney (1991) discussed several basic formulation issues and procedures to solve a cutting problem. Settlement procedures are summarized in a linear programming model. Further, sequencing pattern to cut or allocation patterns, analyzed in a study conducted by Yuen and Richardson (1995) in order to minimize the maximum queue in the process of cutting. Foronda and Carino (1991), Yanasse (1991), Carnieri et al. (1994), Morabito and Garcia (1998) and Parwadi (2009) proposed a heuristic approach in allocating sawmill.

In this study, a two-phase algorithm was developed for the cutting sequencing problem of a furniture manufacturer. In the first phase, a linear integer programming model is used to determine the number of sheets of wood to be cut by each type of pattern. This cutting pattern produced by a special software program. This cutting pattern is one of the input models. The model was tested by using two different objective functions, which is about minimizing waste, and the other is minimize the amount of wood to be cut. The second phase consists of a heuristic scheduling determined by sequence cutting patterns obtained in Phase 1 by considering the maturity date of the order, and taking into consideration the demands of the item.

This paper is organized in the following order: the first part is introduction, the second part presents a two-phase algorithm, the third part of the implementation of the algorithm on a furniture company XYZ Co. and the final section presents conclusions and recommendations.

II. TWO-PHASE ALGORITHM

A. *Phase 1: Integer Linear Programming Models*

At this first pahse, a linear programming model is developed. This model was used to determine the number of hardboards that will be cut by each cutting pattern. Suppose that the number of hardboards that will cut the *i*-th cutting pattern given by the symbol x_i. There are three types of constraints used in this model, namely:

(a) Constraints that ensure that all products demanded by customers will be produced.

$$\sum_{i=1}^{P} \alpha_{ik} x_i \geq d_k, \qquad \forall k \qquad (1)$$

where :

α_{ik} = the number of product type k produced using the cutting pattern-i

d_k = the total demand of product type k

P = the number of cutting pattern

(b) Constraints which considers the capacity of the hardboardss used.

$$\sum_{i=1}^{P} x_i \leq Q \qquad (2)$$

where Q is the available capacity of the hardboards used.

(c) Constraints that ensure nonnegativity integer number for all variables.

$x_i \geq 0$ for all integer i (3)

The objective that will be achieved in this model is to determine the number of sheets of cut blocks in each cutting pattern. This model was tested by applying two different objective functions, but uses the same constraints for both formulations as a comparison.

The first objective function aims to minimize the total waste (called by Model 1), that is

$$Z = \sum_{i=1}^{p} \left(LWH - \sum_{k=1}^{N} l_k w_k h_k \alpha_{ik} \right) x_i \qquad (4)$$

where:

l x w x h = product size (length x width x height)

L x W x H = raw material size (length x width x height)

The second objective function aims to minimize the number of hardboards that will be cut or used (called by Model 2), that is

$$Z = \sum_{i=1}^{p} x_i \qquad (5)$$

Data used in both models are the size of the hardboards, hardboards capacities, the demand for products and various combinations of cutting patterns. If the both above mathematical model is rewritten back, we gained two mathematical models as follows.

Model 1: Mathematical model with the aim of minimizing waste.

Minimize waste:

$$Z = \sum_{i=1}^{p} \left(LWH - \sum_{k=1}^{N} l_k w_k h_k \alpha_{ik} \right) x_i$$

Subject to:

$$\sum_{i=1}^{P} \alpha_{ik} x_i \geq d_k, \text{for } \forall k$$

$$\sum_{i=1}^{P} x_i \leq Q$$

$x_i \geq 0$, integer for $\forall i$.

Model 2: Mathematical model with the aim of minimizing the number of hardboards used.

Minimize the number of hardboards:

$$Z = \sum_{i=1}^{p} x_i$$

Subject to :

$$\sum_{i=1}^{P} \alpha_{ik} x_i \geq d_k \quad for \ \forall k$$

$$\sum_{i=1}^{P} x_i \leq Q$$

$x_i \geq 0$ integer for $\forall i$.

B. Phase 2: Heuristics Scheduling

In the second phase contains about heuristic computation to determine the scheduling of cutting patterns obtained from the Phase 1. This phase also considers the number of demand for each product on each request. Computation of the heuristic scheduling is to follow the EDD (Earliest Due Date) rule. This is done in achieving one goal to minimize delays in production scheduling and delivery in order to meet consumer demand and companies are not penalized due to delay of the consumer.

Initialization:

Input the following values:

x_i = the number of hardboards that will cut by the i-th cutting pattern

α_{ik} = the number of product k produced by the i-th cutting pattern

d_{jk} = the total demand of product k in order-j

Step 1

Sequence the order according to the EDD rule.

Step 2

Write the vector of demand-j using formula:

$$O_j = (d_{j1}, d_{j2}, \ldots, d_{jk}) \qquad (6)$$

Step 3

Compute the weight of the element of demand-j by normalizing the value using formula:

$$w_{jk} = \frac{d_{jk}}{\max_k \{d_{jk}\}} \qquad \forall k \qquad (7)$$

Step 4
Compute the matching score between demand-*j* and pattern-*i* using formula:

$$\mathrm{MS_i} = \sum_{k=1}^{N} \alpha_{ik} w_{jk} \quad \forall \mathrm{i} \tag{8}$$

where N = the number of product type.

Step 5
Select the best pattern θ having the highest machine value MS_i using formula:

$$\mathrm{MS}_\theta = \max_\mathrm{i} \{\mathrm{MS_i}\} \tag{9}$$

Step 6
Determine the required number of hardboards cut by pattern θ for all product *k* using formula:

$$nc_{\theta k} = \begin{cases} d_{jk} / \alpha_{\theta k} & ,\text{if } \alpha_{\theta k} > 0 \text{ and } d_{jk} > 0 \\ 0 & ,elsewhere \end{cases} \tag{10}$$

Step 7
Determine the required maximum number of hardboards cut by pattern θ from among $nc_{\theta k}$ determined in Step 6 using formula:

$$nc_{\theta k*} = \max_k \{nc_{\theta k}\} \tag{11}$$

Step 8
Determine the actual number of hardboards cut by pattern θ using formula:

$$nc_{\theta k*} = \begin{cases} x_\theta & ,\text{if } nc_{\theta k*} > x_\theta , \\ & \quad set\ x_\theta \leftarrow 0 \\ nc_{\theta k*} & ,\text{if } x_\theta \geq nc_{\theta k*} , \\ & \quad set\ x_\theta \leftarrow x_\theta - nc_{\theta k*} \end{cases} \tag{12}$$

Step 9
Update the vector of demand-*j* using formula:

$$ns_k \leftarrow \alpha_{\theta k} nc_{\theta k*} + ns_k \tag{13}$$

$$d_{jk} \leftarrow d_{jk} - ns_k \quad \forall k \tag{14}$$

where ns_k = the number of product *k* in stock. Update the vector of pattern θ by:

$$\alpha_{\theta k} \leftarrow \begin{cases} 0 & ,\text{if } x_\theta = 0 \\ \alpha_{\theta k} & ,elsewhere \end{cases} \tag{15}$$

If $d_{jk} \leq 0$, for all product *k*, go to Step 2, and read the next demand-*j*.

If not, go to Step 4, and select another pattern, while renormalize the element of vector demand-*j* (Step 3).

Step 10
Repeat Step 2 – 9 until all order vectors are exhausted.

III. ALGORITHM IMPLEMENTATION

The proposed two-phase algorithm is implemented in a medium-size furniture manufacturer located in Jakarta Indonesia, named XYZ Company. XYZ Co. is a manufacturing company engaged in timber production. Products produced by the XYZ Co. are wooden sticks that have the length, width and thickness in accordance with consumer demand. Usually the consumer will change the wood products into other products are ready for use. Therefore, XYZ Co. may be said as a company that produces semi-finished goods for consumers, which will be reprocessed into finished goods.

Raw material used by this company is the raw material sheet of wood beams. In this study, the size of the raw materials used in the hardboards has a length of 2000 mm, width 200 mm and 90 mm thick. Whereas the capacity availability of raw material of hardboards sent by the supplier is about 350 m^3 which is equivalent to 9722 units of the hardboards. This study used 5 products derived from the reservation that are product A, B, C, D and E.

In this study, there are 3 demands per product per month. All three demand data shows that:

1. First demand, namely the total number of product demand from the 1st until the 10th of the month.
2. Second demand, namely the total number of product demand from 11 to the 20th of the month.
3. Third demand, namely the total number of product demand from 21 until the end of the month.

Here is a consumer demand for each product that is written in TABLE I.

TABLE I
CONSUMERS' DEMAND FOR EACH PRODUCT (IN MM^3)

Product Type	Demand 1	Demand 2	Demand 3	Total Demand
A	1062	892	937	2891
B	904	867	880	2651
C	672	604	0	1276
D	28	3	33	64
E	43	64	7	114

In this study the date demand 1 < due date demand 2 < due date demand 3.

Determination of various combinations of cutting patterns begins by entering the data size of the product and raw material sheet size blocks are used. Product size and the size of thick slabs and beams have the same units. Units used by the company in the form of units of millimeters.

Variety combination of cutting is done based on two considerations, namely:

1. Based on the width of the raw material compared to the width of the product (code LL). Pattern cutting width viewed from the priorities of raw materials compared with the width of the product, then the pattern of new products

that have counted the number of products that can be cut under the product on the length of the remaining raw materials.

2. Based on the long-term raw material compared with the product (code PP).

Cutting pattern seen from the long priority of raw materials compared with the length of the product, then the pattern of new products that have counted the number of products that can be cut under the product on the remaining width of raw material.

Variety of combination is obtained by using the help of Visual Basic 6.0 software to produce 13 combinations as shown in TABLE II.

TABLE II
VARIETY OF CUTTING COMBINATION

No	Combination Type	A	B	C	D	E	Remaining Volum (mm^3)	Code
1	BA	6	4	0	0	0	5350040	LL
2	AE	6	0	0	0	2	5393336	LL
3	BB	0	8	0	0	0	9445120	LL
4	EB	0	4	0	0	2	9488416	LL
5	E	0	0	0	0	4	9531712	PP
6	CA	6	0	2	0	0	9787880	LL
7	AC	2	0	4	0	0	12529960	PP
8	CB	0	4	2	0	0	13882960	LL
9	EC	0	0	2	0	2	13926256	LL
10	BA	2	4	0	0	0	16931720	PP
11	D	0	0	0	2	0	18092880	LL
12	CC	0	0	4	0	0	18320800	LL
13	AAA	6	0	0	0	0	18627480	PP

From the table above, it can be explained as follows that:

1. Pattern 1 produces a combination of BA with 6 units of product A and 4 units product B so that the remaining volume of 5350040 mm 3; pattern is cut on a width priority basis first.
2. Pattern 2 produces a combination of AE with 6 units of product A and 2 units product E so that the remaining volume of 5393336 mm 3; pattern is cut on a width priority basis first.
3. Pattern 3 produces a combination of BB with 8 units product B so that the remaining volume of 9445120 mm 3; pattern is cut on a width priority basis first.
4. Pattern 4 produces a combination of EB with 4 units product B and 2 units E so that the remaining volume of 9488416 mm 3; pattern is cut on a width priority basis first.
5. Pattern 5 produces a combination of E with 4 units product E so that the remaining volume of 9531712 mm 3; pattern is cut on a length priority basis first.
6. Pattern 6 produces a combination of CA with 6 units of product A and 2 units product C so that the remaining volume of 9787880 mm 3; pattern is cut on a width priority basis first.
7. Pattern 7 produces a combination of AC with 2 units of product A and 4 units product C so that the remaining volume of 12,529,960 mm3; pattern is cut on a length priority basis first.
8. Pattern 8 produces a combination of CB with 4 units of product B and 2 units product C so that the remaining volume of 13,882,960 mm3; pattern is cut on a width priority basis first.
9. Pattern 9 producing a combination of EC with 2 units product C and 2 units product E so that the remaining volume of 13,926,256 mm3; pattern is cut on a width priority basis first.
10. Pattern 10 to produce a combination of BA with 2 units of product A and 4 units product B so that the remaining volume of 16,931,720 mm3; pattern is cut on a length priority basis first.
11. Pattern 11 to produce a combination of D with 2 units products D so that the remaining volume of 18,092,880 mm3; pattern is cut on a width priority basis first.
12. Pattern 12 to get a combination of CC with 4 units products C so that the remaining volume of 18,320,800 mm3; pattern cut by widths priority first.
13. Pattern 13 to produce a combination of AAA with 6 unit product A so that the remaining volume of 18,627,480 mm3; pattern is cut on a length priority basis first.

Phase 1 Integer Linear Programming Model
Model 1

Using the data shown on Table I and Table II and considering the avalable capacity, then Model 1 become:

Objective function :

$$\text{Minimizei } Z = 5350040\,x_1 + 5393336\,x_2 + 9445120\,x_3 + 9488416\,x_4 + 9531712\,x_5 + 9787880\,x_6 + 12529960\,x_7 + 13882960\,x_8 + 13926256\,x_9 + 16931720\,x_{10} + 18092880\,x_{11} + 18320800\,x_{12} + 18627480\,x_{13}$$

Subject to:

$$6x_1 + 6x_2 + 6x_6 + 2x_7 + 2x_{10} + 2x_{13} \geq 2891$$
$$4x_1 + 8x_3 + 4x_4 + 4x_8 + 4x_{10} \geq 2651$$
$$2x_6 + 4x_7 + 2x_8 + 2x_9 + 4x_{12} \geq 1276$$
$$2x_{11} \geq 64$$
$$2x_2 + 2x_4 + 4x_5 + 2x_9 \geq 114$$
$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13} \leq 9722$$
$$x_j \geq 0 \text{ and integer}$$

where :

Z = waste

x_j = the number of hardboards that will be cut by cutting pattern j (j = 1,2,..., 13).

By Using software WinQSB, we get the results as follows:

x_1 = 375, x_8 = 0
x_2 = 1, x_9 = 0
x_3 = 142, x_{10} = 0
x_4 = 4, x_{11} = 32
x_5 = 26, x_{12} = 0
x_6 = 0, x_{13} = 0
x_7 = 319
Z = 8214672000 mm^3
Σx = 899 units.

From Model 1 we obtain that the total waste are 8214672000 mm^3 and the total number of hardboards are 899 units.

Model 2

Using the data shown on TABLE I and TABLE II and considering the avalable capacity, then Model 2 become:

Objective function :

Minimize Z = $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13}$

Subject to :

$$6x_1 + 6x_2 + 6x_6 + 2x_7 + 2x_{10} + 2x_{13} \geq 2891$$
$$4x_1 + 8x_3 + 4x_4 + 4x_8 + 4x_{10} \geq 2651$$
$$2x_6 + 4x_7 + 2x_8 + 2x_9 + 4x_{12} \geq 1276$$
$$2x_{11} \geq 64$$
$$2x_2 + 2x_4 + 4x_5 + 2x_9 \geq 114$$
$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13} \leq 9722$$
$$x_j \geq 0 \text{ and integer}$$

where :

Z = the total number of hardboards

x_j = the number of hardboards that will be cut by cutting pattern j (j = 1,2,..., 13).

By Using software WinQSB, we get the results as follows:

x_1 = 376, x_8 = 0
x_2 = 0, x_9 = 0
x_3 = 143, x_{10} = 0
x_4 = 1, x_{11} = 32
x_5 = 28, x_{12} = 0
x_6 = 0, x_{13} = 0
x_7 = 319
Z = 899 units
Σx = 899 units.

From Model 2 we obtain that the total number of hardboards are 899 units. We see that from these two models we obtain that the total number of hardboards are the same.

Phase 2: Heuristics Scheduling

Heuristics Scheduling of Model 1

i = cutting pattern *i*-th (1, 2, 3, ..., 13)
j = demad *j*-th (1, 2, 3)
k = product type (A, B, C, D, E)
x_i = the number of hardboards that will be cut by cutting pattern *i*-th
α_{ik} = the number of product type k produxced by cutting pattern *i*-th
d_{jk} = the total demand of product type *k* in demand *j*-th

Cutting scheduling to minimize waste shown on the following table.

TABLE III
CUTTING SCHEDULING TO MINIMIZE WASTE

Scheduling No.	Cutting pattern (i)	The number of hardboards that will be cut by cutting pattern *i*-th (x_i)	Total Demand
1	1	226	1
2	7	168	1
3	5	11	1
4	11	14	1
5	3	109	2
6	7	151	2
7	5	15	2
8	11	2	2
9	4	2	2
10	1	149	3
11	3	33	3
12	11	16	3
13	4	2	3
14	2	1	3

The number of hardboards used
= 375 + 1 + 142 + 4 + 26 + 0 + 319 + 0 + 0 + 0 + 32 + 0 + 0
= 899 units

Total waste = (375 x 5350040) + (1 x 5393336) +(142x 9445120) + (4 x 9488416) +(26 x 9531712) + (0 x 9787880) + (319 x 12529960) + (0 x 13882960) + (0 x 13926256) + (0 x 16931720) + (32 x 18092880) + (0 x 18320800) + (0 x 18627480) = 8214672952 mm^3

The percentage of total waste of all the sheets beam used is

$$\frac{\text{Total Waste}}{\sum \text{total sheet beam used x Volum of the sheet beam}} \text{ x } 100\%$$

$$= \frac{8214672952 \text{ mm}^3}{899 \text{ x } (2000\text{x}200\text{x}90) \text{ mm}^3} \text{ x } 100\%$$

= 25.38 %

Heuristics Scheduling of Model 2

The cutting scheduling that minimize the number of hardboards shown on the following table.

The number of hardboards used
=376 + 0 + 143 + 1 + 28 + 0 + 319 + 0 + 0 + 0 + 32 + 0 + 0
= 899 unitS

Total waste
=(376 x 5350040) + (0 x 5393336) + (143 x 9445120) +
(1 x 9488416) +(28 x 9531712) + (0 x 9787880) +
(319 x 12529960) + (0 x 13882960) + (0 x 13926256) +
(0 x 16931720) + (32 x 18092880) + (0 x 18320800) +
(0 x 18627480) = 8214672952 mm^3

TABLE 4
CUTTING SCHEDULING TO MINIMIZE NUMBER OF HARDBOARDS

Scheduling No.	Cutting pattern (i)	The number of hardboards that will be cut by cutting pattern *i*-th (x_i)	Total demand
1	1	226	1
2	7	168	1
3	5	11	1
4	11	14	1
5	3	109	2
6	7	151	2
7	5	16	2
8	11	2	2
9	1	150	3
10	3	34	3
11	11	16	3
12	4	1	3
13	5	1	3

The percentage of total waste of all the sheets beam used is

$$\frac{\text{Total Waste}}{\sum \text{total sheet beam used x Volum of the sheet beam}} \times 100\%$$

$$= \frac{8214672952 \text{ mm}^3}{899 \text{ x } (2000\text{x}200\text{x}90) \text{ mm}^3} \times 100\%$$

= 25.38 %

From these two model formulations obtained that the total waste of raw materials are the same that is 8214672952 mm3, and the percentage of total waste of all the sheets beam used is is equal to 25.38%, but the two cutting scheduling diffrent in term of the number of scheduling and the order of cutting pattern.

TABLE 5
COMPARISON RESULTS OF MODEL 1 AND MODEL 2

Model	Total number of hardboards used (units)
Model 1	899
Model 2	899

The difference results of both computations minimizing waste and minimization the hardboards occurs on the order of scheduling based on the EDD, which model of minimizing waste obtained 14 cutting pattern scheduling sequence and the model of minimize the total number of hardboards used obtained 13 cutting pattern scheduling sequence. Since the results of both formulations are the same, then the decision to apply one of the two model formulations and cutting scheduling handed to the company.

IV. CONCLUSIONS AND RECOMMENDATIONS

In this study, a two-phase algorithm was developed for the cutting sequencing problem of a furniture manufacturer located in Jakarta. In the first phase, an integer linear programming model is used to determine the number of hardboards that will be cut by each pattern type. The model is tested with two different objective functions for comparison. The cutting patterns, generated by a special software program, are among the inputs of this model. The second phase consists of a heuristic that decides on the sequencing of the cutting patterns that are obtained in phase 1 by considering the due date of the orders, and taking into account the demands of the items.

The implementation results were discussed with the planners working in the furniture manufacturer, and they indicated that the proposed algorithm could be employed as a useful tool for the cutting sequencing problem of the company.

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REFERENCES

[1] Carnieri, C., Mendoza, G.A., Gavinho, L.G. (1994), "Solution procedures for cutting lumber into furniture parts", *European Journal of Operational Research*, Vol. 73 No.3, pp.495-501.
[2] Foronda, S.U., Carino, H.F. (1991), "A heuristic approach to the lumber allocation problem in hardwood dimension and furniture manufacturing", *European Journal of Operational Research*, Vol. 54 pp.151-62.
[3] Haessler, R.W., Sweeney, P.E. (1991), "Cutting stock problems and solution procedures", *European Journal of Operational Research*, Vol. 54 No.2, pp.141-50.
[4] Morabito, R., Garcia, V. (1998), "The cutting stock problem in a hardboard industry: a case study", *Computers & Operations Research*, Vol. 25 No.6, pp.469-85.
[5] Parwadi, M. (2009), "An integrated distribution-inventory-production model for billet steel product". *Journal of Business and Management*. Vol. 6 No. 2, pp. 352-360.
[6] Parwadi, M. (2010), "The relationship between production system, business strategy, competitive environmetal and culture organization in manufacture industry". *Journal of Business and Management research media"*. Vol. 9 No. 2, pp. 135-152.
[7] Yanasse, H.H. (1991). "Two-dimensional cutting stock with multiple stock sizes". *Journal of the Operational Research Society*, Vol. 42 No.8, pp.673-83.
[8] Yazgaç T., Rifat G.O. (2004). "A Cutting Sequencing Approach To Modular Manufacturing". *Journal of Manufacturing Technology Management*. 15(1) : 20 – 28.
[9] Yuen, B.J., Richardson, K.V. (1995), "Establishing the optimality of sequencing heuristics for cutting stock problems", European Journal of Operational Research, Vol. 84 pp.590-8.