Kinematic and Dynamic Modeling of a Multi-Fingered robot Hand

Wan Faizura Binti Wan Tarmizi, Irраivan Elamvazuthi and Mumtaj Begam

Abstract— Research on multi-fingered robot hand (MFRH) is being carried out to accommodate a variety of tasks such as grasping and manipulation of objects in the field of industrial applications, service robots, and rehabilitation robots. The first step in realizing a fully functional MFRH is mathematical modeling. In this paper, a MFRH model is proposed based on the biological equivalent of human hand where each links interconnect at the metacarpophalangeal (MCP), proximal interphalangeal (PIP) and distal interphalangeal (DIP) joints respectively. The Kinematic and Dynamic modeling was carried out using Denavit Hartenburg (DH) algorithm and Euler Langrange formula for the proposed MFRH model.

Index Term— Multi-fingered robot hand, Modeling, Robotics, Simulation

I. INTRODUCTION

A robot hand is defined as that can mimic the movements of a human hand in operation. Stable grasping and fine manipulation with the multi-fingered robot hands (MFRH) are playing an increasingly important role in manufacturing and other applications that require precision and dexterity [1]. The dexterous hand for WENDY [2] and DLR II [3] are examples of MFRH. This type of MFRH has advantage that the hand can be used with various types of robot arms because the robot hand has independent structure. On other hand, there are disadvantages. The most serious one is the limitation on size. Most of this type of robot hand has equal to or less than four fingers. Even, those with five fingers are not equal with human hand because they have less number of joints or Degree of Freedom (DOF) [4].

The need for improving the MFRH arises from the desire for handling objects and shapes more effectively. Therefore, mechanical design plays an important role in the development of a MFRH. Simulation eases the design process. Mathematical modeling is an asset to establish simulation.

Mathematical modeling comprising the kinematics and dynamics aspects of MFRH needs to be established. Kinematics is the study of motion without taking into account what causes the motion, whilst Dynamic describe the relationships between different time-based version of motion of the hand, and the motion generating force on the hand [5].

II. LITERATURE REVIEW

David [6] has provided a configuration-space description of the kinematics of the fingers plus-object system for multi-fingered manipulation. Emily [7] has derived kinematics and dynamics equations in the design of an anthropomorphic robotic hand for space operation. Paraspurnan and Shiah [8] have derived kinematics and dynamics equations for biomechanical analysis of human joints. Paraspurnan [9] has improved kinematics derivation for humanoid robot manipulators to be used in the simulation MFRH using virtual reality toolbox. Valentin et al. [10] have given derivation for trajectory planning of a robot manipulator. Kevin and Thurston [11] have derived kinematics analysis of novel 6 DOF parallel manipulator. This kinematic analysis is used for three elbow angles before computing the position and orientation of the top plate. Mina et al. [12] have used inverse kinematics to find the joint of the robot finger. In dynamical model, Yavin [13] has derived the kinematic and dynamic for three-link manipulator. Jonker and Aarts [14] have improved dynamic simulation for the planar flexible link manipulator. Panagiotis and Kostas [15] have improved the kinematics and dynamic to find the position and force of the robot arm in application to teleoperation and orthosis. Ronen et al. [16] have used kinematics and dynamic equations for the planary flexible actuated parallel robot. Aaron [17] has derived forward and inverse kinematic for biologically inspired dexterous robot hand. Ranasamy and Arshad [18] also have derived the equation of kinematic and dynamic to the robot hand simulation using 30 Studio Max and Maya 30. The preliminary results of the study were presented in the ICORAFFS conference proceedings [19].

III. METHODOLOGY

The flowchart in Fig. 1 shows the methodology used in this study for the development of Multi Fingered Robot Hand (MFRH).
It involves the derivation of mathematical model, followed by simulation using the derived mathematical model, development of control algorithm, design and development of hardware and finally testing after total assembly. This paper sets out the framework for the mathematical modeling of kinematic and dynamic equations for the MFRH. The results of other study would be published in the near future.

Mathematical modeling is important in establishing the simulation of a MFRH. In this paper, the mathematical modeling using the Denavit-Hartenburg (DH) algorithm that provides a matrix method to derive the kinematic solution for MFRH is described. The kinematics equation is consisting of forward and inverse kinematics. The forward kinematic solution of a MFRH can be used to determine the position and orientation of the robot hand relative to the robot base coordinate system. It is also computes the joints of the robot hand. The joints of MFRH are referenced according to the biological equivalent; each links interconnect at the metacarpophalangeal (MCP), proximal interphalangeal (PIP) and distal interphalangeal (DIP) joints respectively. The Jacobian and dynamic also are derived to determine the torque and force of the robot hand.

Human hand is a very articulated structure. The high functionality of the human hand is based on the higher degrees of freedom. Human hand has 23 DOF that is provided by 17 joints [19]. If three dimensional movements are taken into consideration, degrees of freedom increase to 29 because of orientation and position variation of the hand. The joint of a multi-fingered robot hand is shown in Fig. 2.

IV. RESULT AND DISCUSSION

Fig. 3 shows the model of the proposed MFRH. In Fig. 3, the fingers are assumed to be index, middle, ring and little fingers.
The frame are named by number according to which they are attached. The convention that was used to locate the frame on the links is known as the D-H convention [20] which is given below:

The \( z \)-axis of frame \( \{i\} \), called \( \{z_i\} \), is coincident with the joint \( i \).

The origin of frame \( \{i\} \) is located where the \( \alpha_i \) perpendicular intersects the joint \( i \) axis.

\( x_i \) points along \( a_i \) in the direction from joint \( i \) to joint \( i+1 \).

Assuming that the frames have been attached to the links according to the D-H convention, the following definitions of the link parameters are valid [20]:

Rotate the frame \( x_{i-1}y_{i-1}z_{i-1} \) about the \( z_{i-1} \) axis through an angle \( \theta_i \).

Translate the current frame \( x_{i-1}y_{i-1}z_{i-1} \) along the current \( z_{i-1} \) axis by \( d_i \) units.

Translate the current frame \( x_{i-1}y_{i-1}z_{i-1} \) along the current \( x_i \) axis by \( \alpha_i \) units.

Rotate the current frame \( x_{i-1}y_{i-1}z_{i-1} \) about the \( x_i \) axis through an angle \( \alpha_i \).

Fig. 4 shows that the finger has four frames with three joints. The first frame also known as the base frame is \( x_0, y_0, z_0 \) and the subsequent frames are assigned as per the fig. starting with \( x_1, y_1, z_1 \) and ending with \( x_4, y_4, z_4 \). The forward kinematic solution of a finger will be assigned using homogenous matrix.

![Fig. 4. Model of one finger](image)

**A. Forward Kinematic**

Forward Kinematic is used to determine the position and orientation of MFRH to determine the position and orientation of the robot hand relative to the robot base coordinate system. The derivation of forward kinematic equation based on Table I.

<table>
<thead>
<tr>
<th>( i )</th>
<th>( \theta_i )</th>
<th>( d_i )</th>
<th>( \alpha_i )</th>
<th>( a_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \theta_1 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>( \theta_2 )</td>
<td>0</td>
<td>( l_1 ) (MCP)</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>( \theta_3 )</td>
<td>0</td>
<td>( l_2 ) (PIP)</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>( l_3 ) (DIP)</td>
<td>0</td>
</tr>
</tbody>
</table>

\[
H_{i=0}^1 = \begin{bmatrix}
  c\theta_i & -s\theta_i & 0 & a_{i-1} \\
  s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & s\alpha_{i-1} & s\alpha_{i-1} d_i \\
  0 & c\theta_i s\alpha_{i-1} & c\theta_i c\alpha_{i-1} & c\alpha_{i-1} d_i \\
  0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
H_0^1 = \begin{bmatrix}
  C_1 & -S_1 & 0 & 0 \\
  S_1 & C_1 & 0 & 0 \\
  0 & 0 & 1 & 0 \\
  0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
H_1^2 = \begin{bmatrix}
  C_2 & -S_2 & 0 & l_1 \\
  S_2 & C_2 & 0 & 0 \\
  0 & 0 & 1 & 0 \\
  0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
H_2^3 = \begin{bmatrix}
  C_3 & -S_3 & 0 & l_2 \\
  S_3 & C_3 & 0 & 0 \\
  0 & 0 & 1 & 0 \\
  0 & 0 & 0 & 1
\end{bmatrix}
\]

And

\[
H_3^4 = \begin{bmatrix}
  1 & 0 & 0 & l_3 \\
  0 & 1 & 0 & 0 \\
  0 & 0 & 1 & 0 \\
  0 & 0 & 0 & 1
\end{bmatrix}
\]

Hence, the forward kinematic for the fingers of robot hand are given by:

\[
H_0^4 = \prod_{i=0}^{n} H_i
\]

We assume that:

\[
\cos \theta_1 = C_1, \cos \theta_2 = C_2, \cos \theta_3 = C_3 \quad \text{and} \quad \cos \theta_4 = C_4
\]

First, find \( H_0^1 = H_0^4 \cdot H_1^2 \cdot H_2^3 \cdot H_3^4 \).
\[
\begin{bmatrix}
C_1 & -S_1 & 0 & 0 \\
S_1 & C_1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
C_2 & -S_2 & 0 & l_1 \\
S_2 & C_2 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

where,
\[
\cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2 = \cos(\theta_1 + \theta_2) = C_{12}
\]
\[
\sin \theta_1 \cos \theta_2 + \cos \theta_1 \sin \theta_2 = \sin(\theta_1 + \theta_2) = S_{12}
\]

Hence,
\[
H_0^3 = \begin{bmatrix}
C_{123} & -S_{123} & 0 & l_1C_1 + l_2C_{12} \\
S_{123} & C_{123} & 0 & l_1S_1 + l_2S_{12} \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

Finally, the derivation of forward kinematic is
\[
H_o^4 = H_o^1H_o^2H_o^3
\]

B. Inverse Kinematic

To find the angle joint of MFRH, the equations are derived using the derivation of Inverse Kinematics [21]. Fig. 5 represents the flexion of angles of one finger where \(l_1\) to \(l_3\) are finger parts and \(\theta_1\) to \(\theta_4\) are the joints in-between and represents the angles.

![Fig. 5. Flexion of angles of one finger](image)

By using the previous forward kinematic homogenous matrices, we assume that,
\[
H_0^3 = \begin{bmatrix}
C_{123} & -S_{123} & 0 & l_1C_1 + l_2C_{12} \\
S_{123} & C_{123} & 0 & l_1S_1 + l_2S_{12} \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

By using the previous forward kinematic homogenous matrices in the equation (5), assume a given orientation in the following:
Based on the result in the homogenous forward kinematic of $H^3_0$,

We assume that:

\[ C_{\phi} = C_{123} \]
\[ S_{\phi} = S_{123} \]
\[ x = l_1C_1 + l_2C_{12} \]
\[ y = l_1S_1 + l_2S_{12} \]

Then, we square both of side of equation $x$ and $y$ above

\[ x^2 = (l_1C_1 + l_2C_{12})^2 \]
\[ y^2 = (l_1S_1 + l_2S_{12})^2 \]

And add both of equation (6) and (7),

\[ x^2 + y^2 = l_1^2C_1^2 + l_1^2S_1^2 + l_2^2C_{12}^2 + l_2^2S_{12}^2 + 2l_1l_2 \]

\[ [C_1(C_1C_2 - S_1S_2) + S_1(C_1S_2 - S_1C_2)] \]

\[ x^2 + y^2 = l_1^2C_1^2 + l_1^2S_1^2 + l_2^2C_{12}^2 + l_2^2S_{12}^2 + 2l_1l_2 \]

\[ [C_1^2 + S_1^2C_2^2 - C_1S_1S_2] \]

To simplify the equation 8, use the formula of trigonometric function below:

\[ C_1 + S_1 = 1 \]

Hence,

\[ x^2 + y^2 = l_1^2 + l_2^2 + 2l_1l_2C_2 \]

To obtain the $\theta_1$, find the $C_2$,

\[ C_2 = \frac{x^2 + y^2 - l_1^2 - l_2^2}{2l_1l_2} \]

Then find $S_2$,

\[ S_2 = \pm \sqrt{1 - C_2^2} \]

Hence,

\[ \theta_1 = \arctan 2(S_2, C_2) \]

\[ x = k_1C_1 - k_2S_1 \]
\[ y = k_1C_1 + k_2S_1 \]

Where, $k_1$ and $k_2$ can be written as $k_1 = l_1 + l_2C_2$, and $k_2 = l_2C_2$

Then, we assume $\alpha = \sqrt{k_1^2 + k_2^2}$ and $\beta = \arctan(k_2, k_1)$

And by using formula [22], the equations of $k_1$ and $k_2$, $k_2$ are

\[ k_1 = r \cos \gamma \]
\[ k_2 = r \sin \gamma \]

Substitute equation (10) and (11) into equation (8) and (9),

\[ \frac{x}{r} = \cos \gamma C_1 + \sin \gamma S_1 \]
\[ \frac{y}{r} = \cos \gamma C_1 + \sin \gamma S_1 \]

Then applying the equation as:

\[ \cos \gamma + \theta_1 = \frac{x}{r} \text{ and } \sin \gamma + \theta_1 = \frac{y}{r} \]

So, to find the $\theta_1$, assume that $\varphi + \theta_1 = \arctan 2(y, x)$

As a result,

\[ \theta_1 = \arctan 2(y, x) - \arctan 2(k_1, k_2) \]

And the final, $\theta_1$ can be solved by using the equations for $S_{\phi}$ and $C_{\phi}$:

Let,

\[ \phi = \theta_1 + \theta_2 + \theta_3 \]

Hence, the equation for $\theta_1$, can be written as:

\[ \theta_1 = \phi - \theta_1 - \theta_2 \]

Or

\[ \arctan 2(S_{\phi}, C_{\phi}) - \theta_1 - \theta_2 \]

C. Jacobian

The next step is Jacobian matrix. Jacobian is a relationship between joint space velocities with task space velocity.

First find the velocity of the frame

\[ x = l_1 \cos \theta_1 + l_2 \cos(\theta_2 + \theta_3) + l_1 \cos(\theta_1 + \theta_2 + \theta_3) \]
\[ y = l_1 \sin \theta_1 + l_2 \sin(\theta_2 + \theta_3) + l_1 \sin(\theta_1 + \theta_2 + \theta_3) \]
\[ \theta = \theta_1 + \theta_2 + \theta_3 \]

Or

\[ x = l_1C_1 + l_2C_{12} + l_3C_{123} \]
\[ y = l_1S_1 + l_2S_{12} + l_3S_{123} \]

Differentiating the equation (14), (16) and (14)

\[ \dot{x} = -l_1S_1\dot{\theta}_1 - l_2S_{12}(\dot{\theta}_1 + \dot{\theta}_2) - l_3S_{123}(\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3) \]
\[ \dot{y} = l_1C_1\dot{\theta}_1 + l_2C_{12}(\dot{\theta}_1 + \dot{\theta}_2) + l_3C_{123}(\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3) \]

The equations of $\dot{x}$ and $\dot{y}$ can be written as:

\[ \dot{x} = -(l_1S_1 + l_2S_{12} + l_3S_{123})\dot{\theta}_1 - (l_2S_{12} + l_3S_{123})\dot{\theta}_2 - l_3S_{123}\dot{\theta}_3 \]
\[ \dot{y} = (l_1C_1 + l_2C_{12} + l_3C_{123})\dot{\theta}_1 - (l_2C_{12} + l_3C_{123})\dot{\theta}_2 - l_3C_{123}\dot{\theta}_3 \]
And \( \partial = \partial_1 + \partial_2 + \partial_3 \)

The velocity is
\[
\dot{\mathbf{V}}_o = o J(\partial)
\]
\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{\theta}
\end{bmatrix} = 
\begin{bmatrix}
-\frac{1}{2} l_1 S_1 \dot{\partial} \\
\frac{1}{2} l_1 C_1 \dot{\partial} \\
-\frac{1}{2} l_2 S_2 \dot{\partial} \\
\frac{1}{2} l_2 C_2 \dot{\partial} \\
-\frac{1}{2} l_3 S_3 \dot{\partial} \\
\frac{1}{2} l_3 C_3 \dot{\partial}
\end{bmatrix}
\]

Hence, \( o J(\partial) \)
\[
\begin{bmatrix}
-\frac{1}{2} l_1 S_1 - l_2 S_2 - l_3 S_3 \dot{\partial} \\
\frac{1}{2} l_1 C_1 + l_2 C_2 + l_3 C_3 \dot{\partial}
\end{bmatrix}
\]

D. Dynamic

Based on the Fig. 5, the Lagrangian method was used to derive the dynamics. In dynamic part, the equations have been derived to find out the torque of MFRH. Referring to the derived forward kinematics,

\[
x = l_1 \cos \partial_1 + l_2 \cos(\partial_1 + \partial_2) + l_3 \cos(\partial_1 + \partial_2 + \partial_3)
\]
\[
y = l_1 \sin \partial_1 + l_2 \sin(\partial_1 + \partial_2) + l_3 \sin(\partial_1 + \partial_2 + \partial_3)
\]

Referring equation forward kinematic above, the angular velocity is computed using Euler langrange formula [23]

\[
\omega = \frac{d\partial}{dt}
\]

The angular velocity is

\[
\omega_1 = \dot{\partial}_1 \\
\omega_2 = \dot{\partial}_2 \\
\omega_3 = \dot{\partial}_1 + \dot{\partial}_2 + \dot{\partial}_3
\]

Then, the linear velocity of mass centre each link of the finger was found using Euler langrange formula [23]:

\[
\dot{x}_1 = -\frac{1}{2} l_1 S_1 \dot{\partial}_1 \\
\dot{y}_1 = \frac{1}{2} l_1 C_1 \dot{\partial}_1 \\
\dot{x}_2 = -\frac{1}{2} l_2 S_2 \dot{\partial}_1 - \frac{1}{2} l_2 S_2 (\dot{\partial}_1 + \dot{\partial}_2) \\
\dot{y}_2 = l_2 C_2 \dot{\partial}_1 + \frac{1}{2} l_2 C_2 (\dot{\partial}_1 + \dot{\partial}_2) \\
\dot{x}_3 = -\frac{1}{2} l_3 S_3 \dot{\partial}_1 - \frac{1}{2} l_3 S_3 (\dot{\partial}_1 + \dot{\partial}_2) - \frac{1}{2} l_3 S_3 (\dot{\partial}_1 + \dot{\partial}_2 + \dot{\partial}_3) \\
\dot{y}_3 = -l_3 C_3 \dot{\partial}_1 - l_3 C_3 (\dot{\partial}_1 + \dot{\partial}_2) - \frac{1}{2} l_3 C_3 (\dot{\partial}_1 + \dot{\partial}_2 + \dot{\partial}_3)
\]

The equation of the velocity linear above should be square and sum of the equations to find \( v_1, v_2 \) and \( v_3 \),

\[
v_1 = \dot{x}_1^2 + \dot{y}_1^2
\]
\[
v_2 = \dot{x}_2^2 + \dot{y}_2^2
\]
\[
v_3 = \dot{x}_3^2 + \dot{y}_3^2
\]

And the link kinetic energy,

\[
K = \frac{1}{2} \sum_k (m_k v_k + \frac{1}{2} l_k_1 \omega_k^2)
\]

\[
K = \frac{1}{2} m_1 (v_1 + \frac{1}{2} l_1 \omega_1)^2 + \frac{1}{2} m_2 (v_2 + \frac{1}{2} l_2 \omega_2)^2 + \frac{1}{2} m_3 (v_3 + \frac{1}{2} l_3 \omega_3)^2
\]

\[
= \frac{1}{2} m_1 \left( \frac{1}{4} \dot{\partial}_1^2 + \frac{1}{4} \dot{\partial}_2^2 + \frac{1}{2} \dot{\partial}_3^2 + \frac{1}{4} \dot{\partial}_1^2 + \frac{1}{2} \dot{\partial}_2^2 + \frac{1}{4} \dot{\partial}_3^2 \right) + \frac{1}{2} l_1 \dot{\partial}_1 \dot{\partial}_2
\]

\[
+ l_2 \dot{\partial}_1 \dot{\partial}_2 + \frac{1}{2} l_3 \dot{\partial}_1 \dot{\partial}_2 + \frac{1}{2} l_3 \dot{\partial}_1 \dot{\partial}_3
\]

The kinetic energy in matrix form is referring this formula [24] below:

\[
K = \frac{1}{2} \left( \begin{array}{c}
\dot{\partial}_1 \\
\dot{\partial}_2 \\
\dot{\partial}_3
\end{array} \right)^T \left( \begin{array}{ccc}
A_{11} & A_{12} & A_{13} \\
A_{21} & A_{22} & A_{23} \\
A_{31} & A_{32} & A_{33}
\end{array} \right) \left( \begin{array}{c}
\dot{\partial}_1 \\
\dot{\partial}_2 \\
\dot{\partial}_3
\end{array} \right)
\]
The mathematical modeling plays an important role in the simulation of multi-fingered robot hand (MFRH). In this paper, the complete derivation of the mathematical modeling comprising the kinematics and dynamics of MFRH was carried out to enable subsequent simulation work. The results will be published in future. Other work such as development of control algorithm and development of MFRH will be addressed in the next phase of study.

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