Analytical Study On Flow Through a Pelton Turbine Bucket Using Boundary Layer Theory

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Abstract**-- Elementary mathematical formulas governing the power developed by the Pelton turbine and design were deduced in early 1883. At that time the principal sources of loss are identified as the energy remaining in water after being discharged from the bucket, The heat developed by impact of water in striking the bucket, The fluid friction of the water in passing over the surface of the bucket, The loss of head in the nozzle, The journal friction the resistance of the air. It was assumed that the effect from of above all the losses were negligible when deriving the Mathematical formula governing the performance of the Pelton wheels. And also it was assumed that the all the water escapes from the bucket with the same velocity. Among the various analytical studies that had been done on Pelton turbine hydraulics less attention has been paid to the friction along the buckets. In this paper the effect of bucket friction was analyzed using Boundary Layer theory.**

I. INTRODUCTION

Hydraulic turbine can be defined as a rotary machine, which uses the potential and kinetic energy of water and converts it into useful mechanical energy. According to the way of energy transfer, there are two types of hydraulic turbines namely impulse turbines and reaction turbines. In impulse turbines water coming out of the nozzle at the end of penstock is made to strike a series of buckets fitted on the periphery of the wheel or runner. Before reaching the turbine Pressure energy of water is converted entirely into kinetic energy. The water leaves the nozzle at atmospheric pressure. The wheel revolves freely in air. Energy transfer occurs due to impulse action. The Pelton turbine and Turgo impulse turbine fall in to this category. In a reaction turbine water enters all around the periphery of runner and the runner remains full of water every time. Only a part of the pressure energy is converted in the guide vanes, into velocity energy. Water acting on wheel is under pressure. A further drop of pressure takes place in the turbine runner. This pressure is greater than atmospheric pressure. The water leaves from the turbine is discharged into the tail race through a draft tube

The draft tube is submerged deep in the tail race. Since the pressure of the water at the inlet to the turbine and outlet are different water should flow in a closed conduit. Therefore casing is necessary for reaction turbines. All the turbine passage is completely full of water and atmospheric air has no access to them. The Francis turbine, Kaplan turbine and propeller turbines are Reaction turbines.

Pelton turbine is an impulse turbine. The runner of the Pelton turbine consists of double hemispherical cups fitted on its periphery. The jet strikes these cups at the central dividing edge of the front edge. The central dividing edge is also called as splitter. The water jet strikes edge of the splitter symmetrically and equally distributed into the two halves of hemispherical bucket. The inlet angle of the jet is therefore between 1° and 3º. Theoretically if the buckets are exactly hemispherical it would deflect the jet through 180°. Then the relative velocity of the jet leaving the bucket would be opposite in direction to the relative velocity of the jet entering. This cannot be achieved practically because the jet leaving the bucket then strikes the back of the succeeding bucket and hence overall efficiency would decrease. Therefore in practice the angular deflection of the jet in the bucket is limited to about 165° or 170°, and the bucket is slightly smaller than a hemisphere in size. The amount of water discharges from the nozzle is regulated by a needle valve provided inside the nozzle. One or more water jets can be provided with the Pelton turbine depending on the requirement.

II. BASIC VELOCITY TRIANGLES

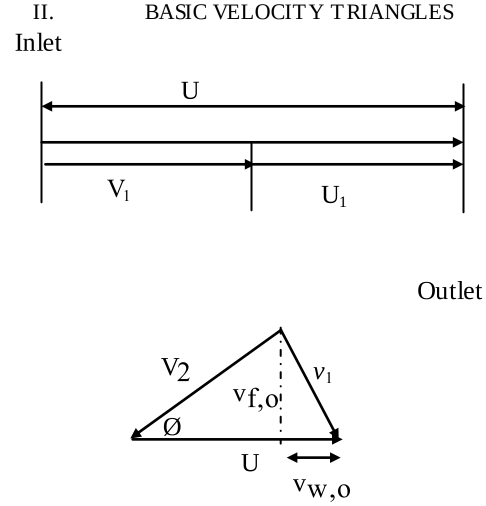

Fig. 1. Basic Velocity Triangles

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Force exerted by water $= \dot{m}(v_{w,i} - v_{w,o})$

work done /Power (P) $= \dot{m}(v_{w,i} - v_{w,o})U_1$

Hydraullic efficiency $= \dfrac{(v_{w,i} - v_{w,o})\dfrac{U_1}{g}}{\dfrac{U^2}{2g}}$

Condition for Maximum hydraulic efficiency of Pelton Wheel

$$v_{w,i} = U = V_1 + U_1$$

$$v_{w,o} = U_1 - v_{r,o} \cos \phi$$

$$\therefore P = \dot{m}(v_{r,i} + v_{r,o} \cos \phi)$$

If there is no frictional resistance along the vanes

$$(v_{r,i} = v_{r,o})$$

$$\therefore P = \dot{m}U_1(U - U_1)(1 + \cos \phi)$$

$$\frac{dP}{dU_1} = \dot{m}(U - 2U_1)(1 + \cos \phi)$$

P to be maximum $\dfrac{dP}{dU_1} = 0$

$$\therefore U = 2U_1$$

In practical situations the friction in the nozzle and friction in the buckets of the Pelton wheel plays an important role. Therefore following empirical relationships are used in designing Pelton Wheels for given conditions.

- Velocity of jet = 0.98 to 0.99 $\sqrt{2gH}$
- Velocity of the runner at pitch diameter = 0.44 to 0.46 $\sqrt{2gH}$
- Angle through which water is deflected in buckets = 165º
- Axial width of buckets = 3.5 to 4 × diameter of the jet
- Number of buckets = $\left(\dfrac{D}{d} + 15\right)$
- Ratio of pitch diameter of runner to jet diameter is not less than 12.

LITERATURE AVAILABLE

Daugherty [3] has analyzed the performance of Pelton wheel in following way.

The net head h supplied to the turbine is estimated as follows.

$$h = h'' + k\frac{V_2^2}{2g} + m\frac{V_2^2}{2g} + k''\frac{V_1^2}{2g} \qquad (1)$$

where h'' is the head converted into mechanical work, the second term represents the energy dissipated from the heat due to internal friction and eddy losses within the runner, the third term is the kinetic energy loss at the discharge, and the fourth term represents the loss in the nozzle of a Pelton wheel.

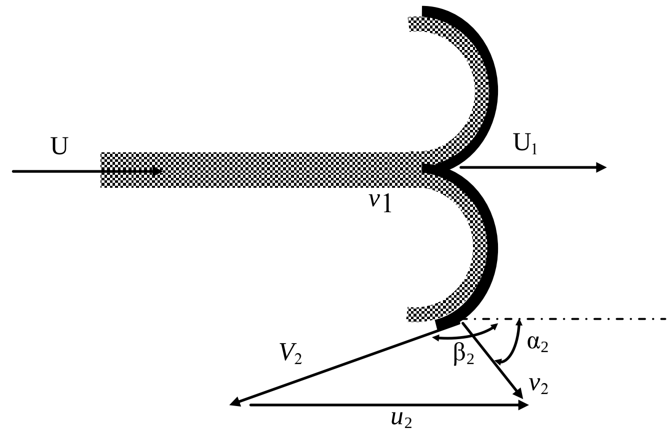

Fig. 2. Schematic diagram of velocity components of flow inside a Pelton Bucket

Head utilized by runner, $h'' = \dfrac{1}{g}(u_1 v_1 - u_2 v_2 \cos \alpha_2)$ -----(2)

Using equation of energy for relative motion of the water in the runner

The total head loss within the runner $= \left(z_1 + \dfrac{v_1^2}{2g} + \dfrac{p_1}{\rho g}\right) - \left(z_2 + \dfrac{v_2^2}{2g} + \dfrac{p_2}{\rho g}\right)$

Using relative velocities and equations 1 and 2

$$\left(z_1 + \frac{(V_1^2 - U_1^2)}{2g} + \frac{p_1}{\rho g}\right) - \left(z_2 + \frac{(V_2^2 - U_2^2)}{2g} + \frac{p_2}{\rho g}\right) = k\frac{V_2^2}{2g}$$

For the Pelton wheel assuming $z_1 = z_2$, $p_1 = p_2$, $u_1 = u_2$

$$V_2 = \frac{V_1}{\sqrt{1+k}} = \frac{v_1 - U_1}{\sqrt{1+k}}$$

Tangential force on the bucket

$$F = \dot{m}(U - u_2 - V_2 \cos \beta_2)$$

$$F = \dot{m}\left(1 - \frac{\cos \beta_2}{\sqrt{1+k}}\right)(V_1 - U_1)$$

Power developed $P = \dot{m}\left(1 - \dfrac{\cos \beta_2}{\sqrt{1+k}}\right)(V_1 - U_1)U_1$

In real situation it cannot be assumed that U=v_1 and always jet is striking the bucket at an angle.

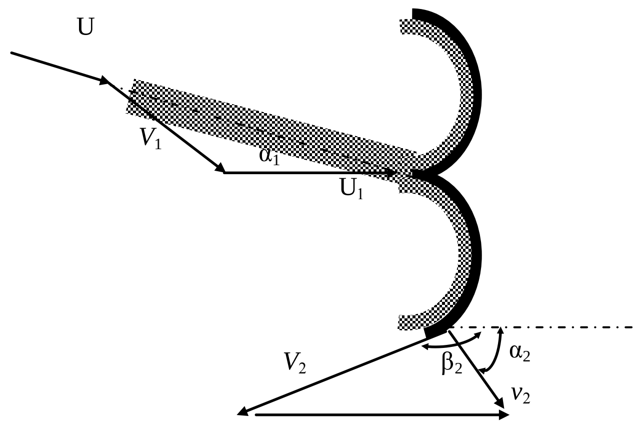

Fig. 3. Schematic diagram of velocity components of flow inside a Pelton Bucket when the Jet striking at angle

Considering all the real situations the equation for power developed by the Pelton turbine is

$$P = \dot{m}\left[U\cos\alpha_1 - x^2U_1 - \frac{x\cos\beta_2}{\sqrt{1+k}}\sqrt{U^2 - 2UU_1\cos\alpha_1 + x^2U_1^2}\right]$$

where $x = \dfrac{r_1}{r_2}$

The value of k was identified as purely empirical value. The value of k depends mainly on the frictional factor along the bucket surface and eddy losses.

An analysis carried out on design of small water turbines by Mahomadd Duralli has also identified two type of losses that occur in the turbine blade passage. One is hydraulic friction loss in the blade and the other one is loss due to flow direction change. The hydraulic friction loss (h_L) is estimated as follows.

$$h_L = f\frac{L}{D_h}\frac{v_2}{2g}$$

Where L is the length of the blade D_h is the hydraulic diameter

$$D_h = \frac{4\times\text{flow area}}{\text{wetted perimeter}}$$

And further it is identified that hydraulic loss through the blade is a function of radius of curvature of the blade, hydraulic diameter and deflection angle of the blade. The main difficulty faced is to find the flow area and the wetted perimeter because analysis of water flow in the bucket is extremely complex. Most of the recent studies that have been done were mainly focused on analysis of flow pattern in the turbine bucket both numerically and experimentally. Because it is revealed that the performance of Pelton turbine is mainly depend on the flow variation along the bucket surface. Unlike in the reaction turbines the hydraulic performance is dynamic due to the unsteady flow in the bucket. So many analytical studies have been done over the years. A very recent comprehensive analysis carried out by the Zh Zhang[5] has done a detail analysis of the control forces in the relative flow of water in the bucket. The relative motion of water in the bucket is affected by the centrifugal, Coriolis and friction forces acting on the free surface flow and it is proven that the effect of centrifugal and Coriolis forces can be neglected.

III. PROBLEM ANALYSIS

The power reduction due to friction force in the Pelton turbine bucket can also be analysed by using the Boundary layer theory. Due to the flow along the bucket surface the boundary layer is formed and retards the velocity of the flow.

The boundary layer equations for the flow pattern in the Pelton wheel involves pressure gradient due to the curvature of the surface. An approximation method for flow with pressure gradient has to be used to solve the problem. Polhausen extended the method of Von Karmens momentum integral equation to cover the flow with pressure gradient choosing a coordinate system in which x denotes the distance measured along the surface and y denotes the normal distance from the surface.

The momentum integral equation can be written as

$$U^2\frac{d\theta}{dx} + \left(2\theta + \delta^*\right)U\frac{du}{dx} = \frac{\tau_0}{\rho} \qquad (3)$$

Assuming the velocity distribution be expressed as a forth degree polynomial equation

$$\frac{u}{U} = a_0 + a_1\eta + a_2\eta^2 + a_3\eta^3 + a_4\eta^4 \quad \text{where} \quad \eta = \frac{y}{\delta}$$

Solving the boundary layer equation with appropriate boundary conditions

$$\frac{u}{U} = \left(2\eta - 2\eta^3 + \eta^4\right) + \frac{\Delta}{6}\left(\eta - 3\eta^2 + 3\eta^3 - \eta^4\right) \qquad (4)$$

Where $\Delta = -\dfrac{\delta^2}{\upsilon}\dfrac{du}{dx}$

$$\delta^* = \delta\left(\frac{3}{10} - \frac{\Delta}{12}\right) \qquad (5)$$

$$\theta = \delta\left(\frac{37}{315} - \frac{\Delta}{945} - \frac{\Delta^2}{90}\right) \qquad (6)$$

$$\tau_0 = \mu\left(\frac{U}{\delta}\right)\left(2 + \frac{\Delta}{6}\right) \qquad (7)$$

The velocity profile of water flow in a Pelton turbine bucket can be assumed to have a velocity similar to the velocity profile of flow over circular cylinder with a free stream velocity U.

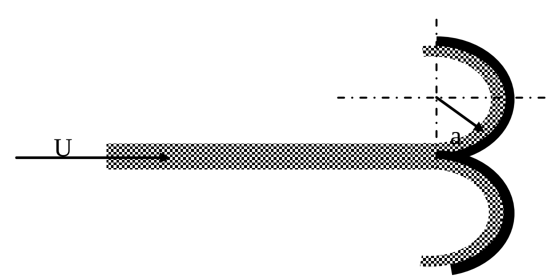

Fig. 4. Flow through Pelton turbine bucket

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The velocity profile can be assumed as follows

$$u = 2U \sin\left(\frac{x}{a}\right)$$

Where a is the radius of the bucket and the distance x is measured along the curved surface.

$$\frac{du}{dx} = \frac{2U}{a}\cos\left(\frac{x}{a}\right)$$

$$\Delta = -\frac{\delta^2}{\upsilon}\frac{2U}{a}\cos\left(\frac{x}{a}\right)$$

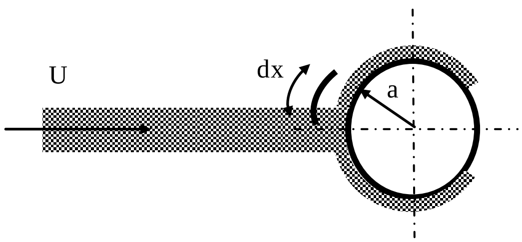

Fig. 5. Flow over circular cylinder

$$\delta^* = \delta\left(\frac{3}{10} + \frac{\delta^2}{12\upsilon}\frac{2U}{a}\cos\left(\frac{x}{a}\right)\right)$$

$$\theta = \delta\left(\frac{37}{315} + \frac{\delta^2}{945\upsilon}\frac{2U}{a}\cos\left(\frac{x}{a}\right) - \frac{1}{90}\left(-\frac{\delta^2}{\upsilon}\frac{2U}{a}\cos\left(\frac{x}{a}\right)\right)^2\right)$$

$$\tau_0 = \mu\left(\frac{U}{\delta}\right)\left(2 - \frac{\delta^2}{6\upsilon}\frac{2U}{a}\cos\left(\frac{x}{a}\right)\right)$$

$$\frac{d\theta}{dx} = -\frac{\delta^2}{945\upsilon}\frac{2U}{a^2}\sin\left(\frac{x}{a}\right) - \frac{1}{90}2\left(\frac{\delta^2}{\upsilon}\frac{2U}{a}\cos\left(\frac{x}{a}\right)\right)\frac{\delta^2}{\upsilon}\frac{2U}{a^2}\sin\left(\frac{x}{a}\right)$$

Substituting these values in momentum integral equation

$$U^2\left(-\frac{\delta^3}{945\upsilon}\frac{2U}{a^2}\sin\left(\frac{x}{a}\right) - \frac{1}{90}2\left(\frac{\delta^3}{\upsilon}\frac{2U}{a}\cos\left(\frac{x}{a}\right)\right)\frac{\delta^3}{\upsilon}\frac{2U}{a^2}\sin\left(\frac{x}{a}\right)\right) +$$
$$\left(2\delta\left(\frac{37}{315} + \frac{\delta^2}{945\upsilon}\frac{2U}{a}\cos\left(\frac{x}{a}\right) - \frac{1}{90}\left(-\frac{\delta^2}{\upsilon}\frac{2U}{a}\cos\left(\frac{x}{a}\right)\right)^2\right) + \delta\left(\frac{3}{10} + \frac{\delta^2}{12\upsilon}\frac{2U}{a}\cos\left(\frac{x}{a}\right)\right)\right)U\frac{2U}{a}\cos\left(\frac{x}{a}\right) =$$
$$\frac{\mu}{\rho}\left(\frac{U}{\delta}\right)\left(2 - \frac{\delta^2}{6\upsilon}\frac{2U}{a}\cos\left(\frac{x}{a}\right)\right) \tag{8}$$

Since δ is small neglecting δ^3 and higher order terms equation (6) can be simplified to

$$2\delta\frac{37}{315} + \frac{3}{10}\delta U\frac{2U}{a}\cos\left(\frac{x}{a}\right) = \frac{\mu}{\rho}\left(\frac{U}{\delta}\right)\left(2 - \frac{\delta^2}{6\upsilon}\frac{2U}{a}\cos\left(\frac{x}{a}\right)\right)$$

$$A\delta + B\delta\cos\left(\frac{x}{a}\right) = \frac{C}{\delta} - D\delta\cos\left(\frac{x}{a}\right)$$

where $\mathrm{A} = \frac{2 \times 37}{315}$,

$\mathrm{B} = \frac{3}{10} \times \mathrm{U} \times \frac{2\mathrm{U}}{\mathrm{a}}$,

$\mathrm{C} = 2 \times \frac{\mu}{\rho}\mathrm{U}$,

$\mathrm{D} = 2 \times \frac{\mu}{\rho}\frac{1}{6\upsilon} \times \frac{2\mathrm{U}^2}{\mathrm{a}}$

$$\delta\left(\mathrm{A} + \mathrm{B}\cos\left(\frac{x}{a}\right) + D\cos\left(\frac{x}{a}\right)\right) = \frac{\mathrm{C}}{\delta}$$

$$\delta^2 = \frac{\mathrm{C}}{\left(\mathrm{A} + \mathrm{B}\cos\left(\frac{x}{a}\right) + D\cos\left(\frac{x}{a}\right)\right)} \tag{9}$$

$$\delta^2 = \frac{\mathrm{C}}{\left(\mathrm{A} + F\cos\left(\frac{x}{a}\right)\right)} \tag{10}$$

where $\mathrm{F} = \mathrm{B} + \mathrm{D}$

by equation (8) and (5)

$$\tau_0 = 2\mu U\frac{\sqrt{\mathrm{A} + F\cos\left(\frac{x}{a}\right)}}{\sqrt{C}} - \frac{D\sqrt{C}\cos\left(\frac{x}{a}\right)}{\sqrt{\mathrm{A} + F\cos\left(\frac{x}{a}\right)}}$$

Power dissipated due to friction

$$P = \int_0^L\left(2\mu U\frac{\sqrt{\mathrm{A} + F\cos\left(\frac{x}{a}\right)}}{\sqrt{C}} - \frac{D\sqrt{C}\cos\left(\frac{x}{a}\right)}{\sqrt{\mathrm{A} + F\cos\left(\frac{x}{a}\right)}}\right)U_1\cos\left(\frac{x}{a}\right).l.dx \tag{11}$$

The integration of the equation (11) gives the power dissipated due to both the direct friction and pressure change.

CONCLUSION.

It is observed that unlike an ideal fluid flow, when flow over a relatively fixed body, the fluid does not slide over the body, but sticks to it, forming boundary layer. The flow gets retarded due to the boundary layer effect. The formation of the boundary layer and the thickness of the boundary layer depends on the surface roughness of the body of which flow is passing through. Hence the power dissipation of the turbine bucket due to the surface roughness can be calculated using the boundary layer analysis. When the water flow inside the Pelton turbine is considered the power loss can be occurred due to both the friction in the bucket and the change of pressure along the flow path.

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NOTATIONS

c_f	Coefficient of friction
D	Diameter of the jet
D_h	Hydraulic diameter
U	Velocity of the jet
U_1	Velocity of the Pelton runner
V	Relative velocity of water
v_1	Absolute velocity of water at the entrance to the bucket
V_1	Relative velocity of water at the entrance to the bucket
v_2	Absolute velocity of water at the exit from the bucket
V_2	Relative velocity of water at the exit from the bucket
$v_{f,o}$	velocity of flow at outlet
$v_{w,o}$	velocity of whirl at outlet
$v_{f,i}$	velocity of flow at inlet
$v_{w,o}$	velocity of whirl at inlet
r	Radius of the bucket
x	Distance measured along the flow path
β	Blade angle
δ	Boundary layer thickness
δ^*	Displacement thickness
θ	Momentum thickness
τ_0	Shear stress