# Reliability Evaluation for a Multi-State System Subject to Imperfect Repair and Maintenance

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***Abstract*— Effective maintenance management is essential to reduce the adverse effect of equipment failure to operation. This is accomplished by accurately predicting the equipment failure such that appropriate actions can be planned and taken in order to minimize the impact of equipment failure to operation. This paper presents a development of model based on Markov process for a degraded multi-state system to evaluate the system performance. The system degradation was quantified by five distinct level of system's production output ranging from perfect functioning state to complete failure with zero output. At any point in time, the system can experience Poisson failure from any state upon which an imperfect repair will be performed while imperfect preventive maintenance will be performed at the last acceptable state as indicated by minimum acceptable production output. This research explored a method of estimating of transition matrix for the five state Markov process by utilizing production output data. The results indicate the applicability of Markov where comparison with traditionally binary model is presented.**

***Index Term*— Multi-State system reliability, Markov process, imperfect repair.**

## I. INTRODUCTION

Effective maintenance management is essential and critical as a way to reduce the adverse effect of equipment failures and to maximize equipment availability. The increase in equipment availability means higher productivity and thus higher profitability provided that the maintenance optimization does include the cost factor. This has lead to increase research interest in the subject of optimizing maintenance management. It is estimated that 15% to 45% of total production cost are attributed to maintenance cost with 30% of total manpower involvement [1]. This is significant; however, the consequence of an inefficient maintenance management is far beyond the direct cost of maintenance although not easily quantifiable. The maintenance's high cost and low efficiency is one of the last cost saving frontier for companies to improve profitability [11] The current research will be focusing on the development of performance evaluation model for repairable equipment subjected to degradation which, in time, reduces the ability of the system to perform its intended function. A repairable system is defined as a system which can be restored to satisfactory working condition by repairing or replacing the damaged components that caused the failure to occur other than replacing the whole system [20]. Performance model would include the evaluation of system reliability as well as system availability with respect to time. The degradation process, if left unattended, will often lead to degradation failure [14]. The degradation can be caused by a myriad of factors including variable operating environment, fatigue, failures of non-essential components and random shocks on the system [16]

## II. BACKGROUND

Traditionally, reliability analysis of repairable system depends upon the assumption that the system can be in a binary state; either fully working conditions or complete failures. With the assumption, numerous approaches, methodologies and models have emerged to predict the reliability of repairable systems corresponding to different repair assumptions. The models include variations of perfect renewals process which assumes perfect repair and non-homogenous Poisson process (NHPP) for minimal repair assumption as discussed in literatures including [8], [12] and [4]. Still, another model called generalized renewal process (GRP) with the assumption that the repair process is in between perfect repair and minimal repair as proposed by [6] and further researched by Yanez, Joglar and Modarres in [21], V. Krivtsov [7] and Weckman et al in [19] to name a few.

However, there are cases as mentioned by researchers such as Soro, Nourelfath and Ait-Kadi [16], Donat, et al.[3] and Ramirez-Marquez and Coit [15] that binary assumption failed to characterize actual system reliability behavior. In these cases, analysis using multi-state system (MSS) assumption is found to be more appropriate. MSS is defined as system that can have a finite number of performance rates with various distinguished level of efficiency [9]. Typical systems where MSS has been applied successfully are in the area of water distribution [13], telecommunication, oil and gas supply system and power generation and transmission [15]. This is due to the fact that there are distinct degradation phases for the system prior to complete failure which is evident from different levels of production outputs. Common methods in accessing the performance of MSS are based on four different approaches: Extension of Boolean models to the multi-valued case, the stochastic process (Markov and semi-Markov), the universal generating function and the Monte-Carlo simulation techniques [9]. Each approach has advantages and disadvantages depending on the system understudy.

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Present research will be focusing on applying Markov process to an absorption chiller system subject to imperfect repair and imperfect preventive maintenance. Imperfect maintenance means that after repair the MSS will be in intermediate state between prefect functioning state and current state. Markov process was chosen to model the system due to its versatility which can be used for finite number of states and different assumptions of repair. The primary advantage of Markov process relies on its ability to describe graphically and mathematically convenient form the time dependent transitions between the system states.

## III. METHODOLOGY

### A. *Markov Chain Process*

The Markov chain is a discrete-time stochastic process where the conditional probability of any future events is only dependent upon current state and is independent of past history. This can be expressed mathematically [13]:

$$P(X_{t+1}=i_{t+1}|X_t=i_t, X_{t-1}=i_{t-1},\dots,X_0=i_0)$$
$$=P(X_{t+1}=i_{t+1}|X_t=i_t) \qquad (1)$$

And as Markov chain assumes that the conditional probability does not change with time and is independent of *t* for all states *i* and *j*

$$P(X_{t+1}=j|X_t=i)=p_{ij} \qquad (2)$$

where $p_{ij}$ = transitional probability from state *i* at time *t* to state *j* at time *t+1*.

Assuming that the Markov chain has *m* number of states, the transition probably can be expressed as an *m x m* matrix which normally known as transition matrix , ***P*** with the characteristics given as follows:

$$P=\begin{bmatrix} P_{11} & \cdots & P_{1m} \\ \vdots & P_{22} & P_{2m} \\ P_{m1} & \cdots & P_{mm} \end{bmatrix} \qquad (3)$$

and

$$\textstyle\sum_{j=1}^{m} P_{ij}=1 \quad \text{for } i = 1,2, \dots, m \qquad (4)$$

Based on the Chapman-Kolmogorov equation, the probability of system moving from state *i* to state *j* after *n* periods can be calculated by multiplying the matrix by itself *n* times. Thus, assuming that $Q^{(0)}$ is the initial state vector, then [2],

$$Q^{(n)} = Q^{(0)}P^n \qquad (5)$$

The calculation for availability and reliability can be done once the probability at each state is determined. Assuming *G(t)* is the performance rate with respect to time and *W* is the constant demand for the system, the availability of the system is at the state where performance rate is greater than demand, that is:

$$A(t)=Pr(G(t)\geq W) \qquad (6)$$

Reliability function, *R(t)*, on the other hand can be calculated by assuming that state 5, which is the complete failure state, is the absorbing state and thus, when the system enters state 5, it never leaves [16]. Thus,

$$R(t)=1-P_5(t) \qquad (7)$$

where $P_5(t)$ is the probability of the system to be in state 5.

### B. *Model Development*

The model will be looking at the performance of an absorption chiller in the form of total refrigeration ton hour (RTh) per day as shown in Fig. 2. Based on RTh output and as observed in the time series plot, there are a number of distinct states that the system can be at any one time depending on the level of daily production. This can be further validated by one way analysis describing the five different states as shown in Fig. 3. The result further substantiates the need to look at the system performance analysis as an MSS compared to using traditional binary reliability modelling framework.

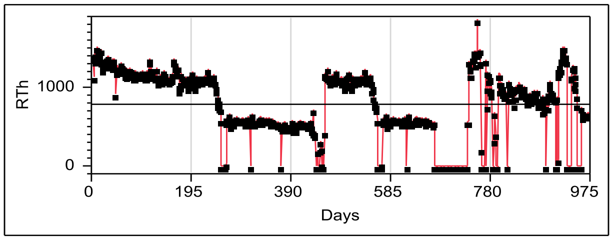

Fig. 1. Daily performance trend for Absorption chiller

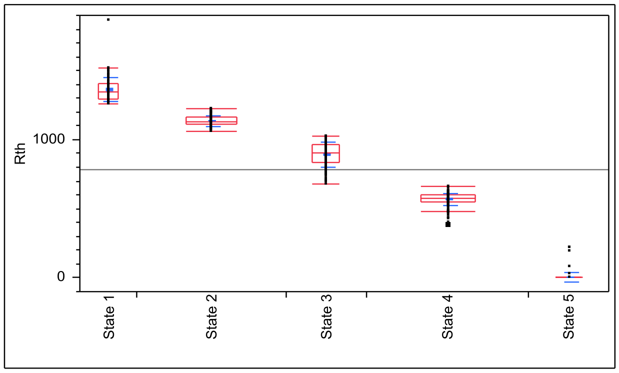

Fig. 2. Absorption chiller discrete states based on daily performance

The limit values for each state are as given in table I

TABLE I

LIMITS FOR EACH STATE

| State | Minimum RTh | Maximum RTh |
|---|---|---|
| 5 | 0.0 | 222.6 |
| 4 | 223.5 | 659.4 |
| 3 | 660.6 | 1025.0 |
| 2 | 1025.0 | 1224.2 |
| 1 | 1225.1 | 2000.0 |

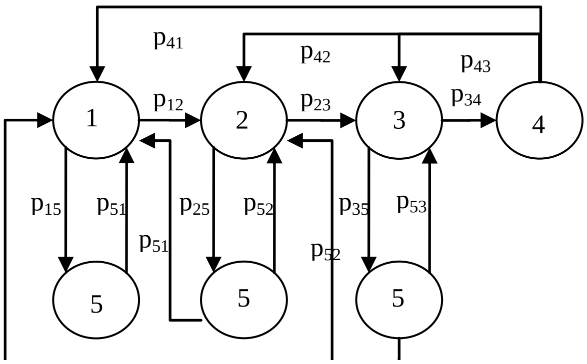

Fig. 3. Absorption chiller state transition diagram

Based on the analysis, there are five distinct states for the selected system with state 1 being the highest performing state and state 5 being total failure with zero output.

### *Model Assumptions*

1. The system may have a number of discrete states depending upon the performance rates which can vary from perfect functionality to complete failure
2. The system may fail randomly from any operational states and will be immediately repaired. The repair can be minimal or imperfect repair
3. All transition rates are constant and sojourn time at each states is exponentially distributed
4. The demand for each system is constant.

### *System description*.

The system state transition diagram is as shown in Fig. 4. With the assumption that the system is initially in the highest performing states as depicted by the highest output in state 1. This is called nominal state. As time progress, the system can either degrade to the output performance of state 2 with probability of $P_{12}$ or can randomly fail with probability $P_{15}$ following Poisson failure which occurs at an instant. If the system fails, it will be repaired minimally returning the system to previous state just before failure. The probability of minimal repair is given as $P_{51}$. When the system reaches it second degraded state (state 2), it can go either to the subsequent degradation state (state 3) or abruptly fails with probabilities $P_{23}$ or $P_{25}$ respectively. If failure does occur, the system will be imperfectly repaired returning the system to either state 1 or state 2. This process continues until the system reaches the last acceptable state (state 4) at which point, the preventive maintenance will be performed. As the assumption is imperfect preventive maintenance, the system can be restored to as good as new or as good as old or the in-between states with probabilities of $P_{41}$, $P_{42}$ and $P_{43}$.

### *Estimation of Transition Probabilities*.

The estimation of transition probabilities is based on performance data. First step is to access the state the system is in for each day based on the maximum and minimum output for each state. The limits for each state are determined based on quantile plots as shown in Fig. 3. If the output of that day falls between the maximum and minimum, say for state 3, then the system will be in the mentioned state for the rest of the day. This was done for the rest of 975 days of available data on production output. Next is to calculate the number of transitions between states. Let $N_{ij}$ is the total number of transitions from state *i* to *j* then

$$P_{ij} = \frac{N_{ij}}{\sum_{i=1}^{5}\sum_{j=1}^{5} N_{ij}} \tag{8}$$

This method of calculation is applied for all the 5 states and the snapshot of the result is as shown in Table I.

TABLE II
CALCULATION RESULTS FOR STATE 5

| Transition | Count | Probability |
|---|---|---|
| 5-1 | 2 | 0.01 |
| 5-2 | 2 | 0.01 |
| 5-3 | 5 | 0.03 |
| 5-4 | 10 | 0.07 |
| 5-5 | 128 | 0.87 |
| Total | 147 | 1 |

The values for all transition probabilities are as shown in Table II.

TABLE III
TRANSITION PROBABILITIES FOR ALL STATES

| | States | | | | |
|---|---|---|---|---|---|
| States | 1 | 2 | 3 | 4 | 5 |
| 1 | 0.82 | 0.15 | 0.0 | 0.0 | 0.03 |
| 2 | 0.0 | 0.94 | 0.06 | 0.0 | 0.0 |
| 3 | 0.0 | 0.0 | 0.9 | 0.06 | 0.04 |
| 4 | 0.0 | 0.0 | 0.0 | 0.94 | 0.06 |
| 5 | 0.01 | 0.01 | 0.03 | 0.07 | 0.88 |

## IV. RESULTS AND DISCUSSION

### A. *State probabilities*

One of the objectives of multi state system analysis is to predict the probabilities of the system to be for each state. With the assumption that the system is in state 1 at time $t=0$, then based on equation (5), the state probabilities can be calculated. The results for state probabilities for 200 days of operation are as depicted in Fig. 5.

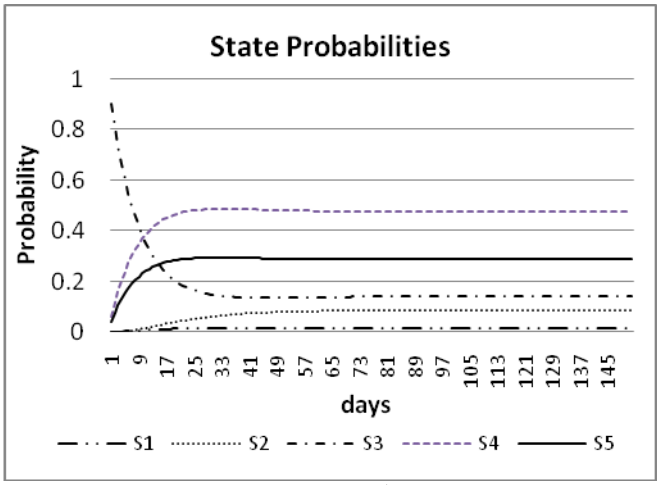

Fig. 4. Probability plot for each state

As can be seen from the fig. above, the state probabilities reach a steady state at around 60 days of operation. This means, at this point in time, it does not matter at which state the initial point is, the system will end up at the at the same state as shown in the fig. above. As the probabilities are no longer dependent on time, the Markov process is said to be homogenous (Lisnianski and Levitin, Multi-State system reliability. Assessment, Optimization and Applications 2003). With the model, maintenance planner would be able to evaluate different maintenance scenario predict the performance of MSS. Fig. 7 shows the results in terms of reliability performance for different preventive maintenance (PM) action with the assumption that the probability is the same. Major PM indicates perfect maintenance bringing the system to be as good as new while minor PM indicates minimal maintenance. As indicated in the fig., as expected, there is an improvement in reliability as the PM action is moved from minimal to imperfect repair to perfect repair.

### *B. Reliability*

Reliability function of the multi-state system understudy can be determined by finding out the probability that the system enters failure state which in this case, state 5 with the assumption that state 5 is absorbing state. This means that whenever the system enters the state it never leaves and this can be achieved by assuming all the probabilities of leaving state 5 are zeros. In other words, the probabilities $P_{51}$, $P_{52}$, $P_{53}$ and $P_{54}$ are all assumed to be zero. Then the reliability is calculated based on equation (6).

For verification on calculated reliability results, a lifetime data analysis with binary model was performed on similar system. Table III shows the time to failure and the frequency. Failure is defined as the occurrence of entering state 5 or having zero output.

TABLE IV

TIME TO FAILURE FOR AN ABSORPTION CHILLER

| Time to failure (days) | Frequency |
|---|---|
| 2 | 2 |
| 3 | 4 |
| 11 | 1 |
| 12 | 1 |
| 16 | 2 |
| 26 | 1 |
| 46 | 1 |
| 47 | 1 |
| 53 | 1 |
| 59 | 1 |
| 67 | 1 |
| 94 | 1 |
| 105 | 1 |

The system failure data was model following a Weibull distribution with parameters shape factor, $\beta$=0.856 and scale factor, $\eta$=29.19 days. The cumulative distribution function is as shown in equation 9. Comparisons of reliability results from both methods are shown in Fig. 6.

$$F(t) = 1 - \exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right] \tag{9}$$

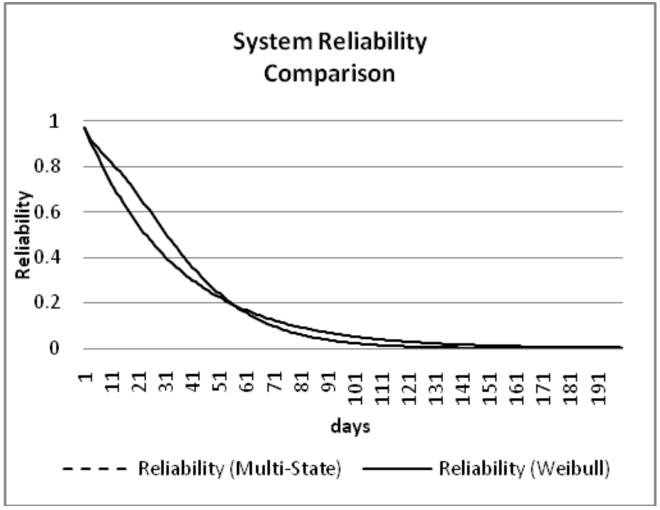

Fig. 5. System reliability comparison

As can be observed from Fig. 6, the reliability calculation results show a close comparison with traditional binary model.

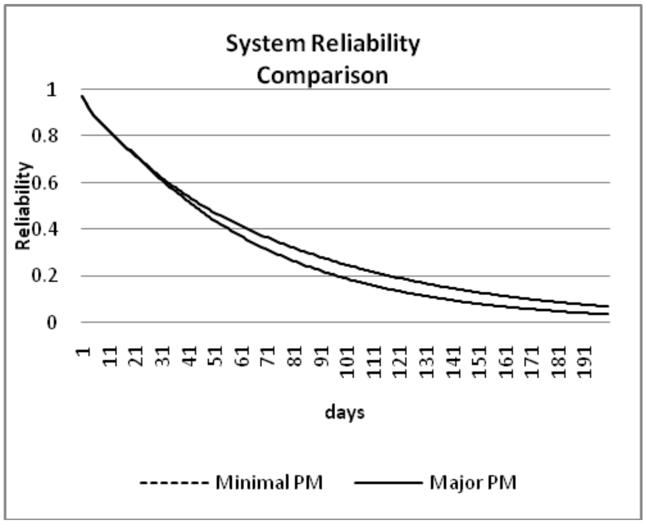

Fig. 6. Impact of different PM assumption to reliability

## V. CONCLUSION AND FURTHER RESEARCH

The current work managed to prove the applicability of Markov process in determining the performance of an MSS which in this case is an absorption chiller, subject to imperfect preventive maintenance and repair. The methodology of calculating transition probability based on daily production data is also provided. Based on the results, the proposed model could be used in practical situation when there is a need to assess the impact of different maintenance and repair actions to MSS performance.

Even though the results are indicating close relevance with traditional method, the proposed model can be further improved by including the variable distribution of sojourn time. This is due to the inherent constraint of Markov process that the state sojourn time needs to be exponentially distributed which is not necessarily true. If the distribution is far from exponential, the results from model will be biased. A more appropriate process will be a semi-Markov which would allow any distribution of sojourn time [3,18] which will be further investigated.

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