Dispersive Study on Tsunami Propagation of The Indian Ocean Tsunami, 26 December 2004

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Abstract-- The Indian Ocean Tsunami (IOT) 26 December 2004 is the most devastating tsunami recorded in history causing over 200,000 deaths and millions homeless people, uncounted property and infrastructure damaged along the coasts of Indonesia, Thailand, Sri Lanka, and the Maldives. Many lessons can be inferred for this case, one of interesting phenomena is the dispersive effect on tsunami propagation is remarkable. Numerical simulations of IOT are conducted here to investigate the effect of dispersion by using weakly nonlinear dispersive Boussinesq (WNB) model and nonlinear shallow water (NLSW) model. Predictor-corrector scheme is used for time integration and high-order finite difference schemes are used for spatial derivatives of the model equations. Simulation results of the two models are compared each other and against recorded data from selected locations to study the dispersive effect. The dispersion effect reduced over than 20% of maximum wave height at some areas of maximum deep water. Very well agreements between WNB and NLSW model at the east areas of tsunami source are obtained by model simulations. However, the NLSW model is very attractive and quite reliable for practical purpose because it has low computation cost and gives consistent results. General features of tsunami wave patterns show a good agreement compared to observations data. Tsunami arrival time and maximum runup are also well predicted by the models.

Index Term-- Boussinesq equations, dispersive, numerical models, nonlinear, tsunami, runup

I. INTRODUCTION

TSUNAMI is very common coastal wave problems in earthquake-prone countries, causing devastating damages in coastal areas. The Indian Ocean Tsunami (IOT) on 26 December 2004 is recorded as highest tsunami. Triggered by tectonic earthquake, the tsunami waves are generated by a complicated bottom uplift/downlift with multiple components of amplitude and frequency. The tsunami was globally propagated over the world ocean as trans-oceanic tsunami propagation. Very complicated structures of spatial and temporal waves are observed by many researchers. One of many lessons from the IOT event is the remarkable of dispersive effect on tsunami propagation. Kulikov in [7] reported the dispersion effect of tsunami waves in the Indian Ocean based on wavelet analysis of recorded satellite data. It is indicated that for trans-oceanic tsunami propagation, dispersion effect could be significant factor for prediction of maximum amplitude. In the coastal area, tsunami waves interact with very complicated bathymetry so the nonlinear combination will affect the profile of tsunami. Hence, model equations which include both dispersive and nonlinear terms are needed for better estimation.

Preliminary results of dispersive numerical model of IOT has been reported in [11] based on Boussinesq-type equations. More detail study of dispersion effect for IOT has been conducted by comparing numerical simulation results of tsunami model based on nonlinear shallow water, nonlinear Boussinesq and the full nonlinear Navier-Stokes in [4]. Discussion the dispersion effect for IOT event has also been reported in [3]. The dispersion effect is noticed at the south-west direction, while at the east part tsunami is essentially non-dispersive [3].

This study is done by reproduce simulation of IOT event using two different model equations i.e.: dispersive and non-dispersive wave models as tools to study the dispersion effect of IOT at the initial stage (up to 3 hours tsunami propagation). The model simulation results are compared each other and against observations data. Better understanding and prediction of tsunami propagation and runup are important for tsunami warning system and for evacuation of peoples when tsunami occurred.

II. MODEL EQUATIONS AND NUMERICAL SOLUTIONS

A. Model Equations

Two sets of model equations are used here i.e.: nonlinear shallow water (NLSW) equations and extended weakly nonlinear Boussinesq-type (WNB) equations derived by Nwogu in [9]. Time-dependent of water depth (bottom) terms are included to the models based on derivation of Lynett and Liu in [8]. By including the time-dependent water depth, the models could be implemented to simulate tsunami generation caused by tectonic plate motions, earthquakes and underwater landslides. The sets of model equations contain equation for conservation of mass and momentum conservation. The model equations are taken in the following form

\[ \frac{\partial \eta}{\partial t} + \nabla \cdot [(h + \eta) \mathbf{u}] = 0 \]

\[ \frac{\partial u}{\partial t} + \mathbf{u} \cdot \nabla (h + \eta) = \frac{1}{\rho} \left( \frac{1}{2} \nabla^2 h + \frac{1}{2} \nabla (\nabla \cdot \mathbf{u}) + \frac{1}{2} \nabla (\nabla \cdot \mathbf{u}) + \frac{1}{2} \nabla (\nabla \cdot \mathbf{u}) \right) \]

\[ \mathbf{F}_b - \mathbf{F}_w = 0 \]

where \( h \) is the still water depth, \( \eta \) is free surface elevation, \( g \) is the gravitational acceleration, and \( \mathbf{F}_b \) is time dependent water depth. Subscript \( t \) denotes partial derivative with respect to time. Two-dimensional vector differential operator \( \nabla \) is
Equations (1) and (2) are general form of model equations used here. Hence, the two set of model equations can be treated using the same numerical algorithm. Setting variables \( \gamma_1 = 1 \) and horizontal velocity vector \( \mathbf{u} = (u, v) \) as velocity at an arbitrary level, \( \mathbf{z} \), reduces the model equations to WNB equations in which \( \mathbf{z} \) is recommended to be evaluated at \( \mathbf{z} = -0.531 h \) (see [9] and [13]). Then setting \( \gamma_1 = 0 \) and using depth averaged horizontal velocity vector \( \mathbf{u} = (\overline{u}, \overline{v}) \) i.e. by taking \( \mathbf{z} = 0 \) reduces the model equations to NLSW equations.

The \( \mathbf{F}_b \) and \( \mathbf{F}_{br} \) terms in (2) are additional terms to accommodate bottom friction and energy dissipation caused by breaking waves, respectively. The bottom friction terms are given in quadratic formula. Although the friction coefficient should be a function of bottom roughness and velocity profile but a simple constant friction coefficient is used in this study. Eddy viscosity formula is used to model the turbulent mixing and energy dissipation caused by breaking waves. Treatment of wave breaking is similar to the eddy viscosity-type formula of [2] and [5].

**B. Numerical Solution**

In order to eliminate the error terms into the same form of dispersive terms in the WNB model equations, fourth-order accuracy of numerical scheme for time stepping and first-order spatial derivatives are used [12]. High order predictor-corrector scheme is used for time stepping, employing third order time explicit Adam-Bashforth scheme as predictor and fourth order Adam-Moulton implicit scheme as corrector step. The corrector step must be iterated until a convergence criterion is satisfied. The system equations are written in a form that makes convenient for application of high-order time stepping procedure. Hence, in Cartesian coordinate system, (1) and (2) are written in the following form

\[
\eta_i = E(\eta_i, u, v) + \left[ E_i(\overline{h}) \right]_i,
\]
\[
U_i = F(\eta_i, u, v) + \left[ F_i(v) \right]_i,
\]
\[
V_i = G(\eta_i, u, v) + \left[ G_i(u) \right]_i
\]

\[
E(\eta, u, v) = -\left( Hu \right)_x - \left( Hv \right)_y
- \left[ a_1 h^3 (u_{xx} + v_{xx}) + a_2 h^2 \left( (hu)_{xx} + (hv)_{xx} \right) \right]_i
- \left[ a_1 h^3 (u_{yy} + v_{yy}) + a_2 h^2 \left( (hu)_{yy} + (hv)_{yy} \right) \right]_i
\]

\[
E_i(\overline{h}) = -\overline{h} - \left( a_1 h^2 \overline{h}_x \right)_x - \left( a_2 h^2 \overline{h}_y \right)_y
\]

\[
F(\eta, u, v) = -g \eta_i - (u^2)_x - v u_x - F_b + F_{br}
\]

\[
F_i(v) = h \left[ b_1 hv_{yy} + b_2 (hv)_{yy} + b_3 \overline{h} \right]
\]

\[
G(\eta, u, v) = -g \eta_i - u v_x - (v^2)_y - G_b + G_{br}
\]

\[
G_i(u) = h \left[ b_1 hu_{xx} + b_2 (hu)_{xx} + b_3 \overline{h} \right]
\]

\[
U = u + b_1 h^2 u_{xx} + b_2 (hu)_{xx}
\]

\[
V = v + b_1 h^2 v_{yy} + b_2 (hv)_{yy}
\]

where \( H = h + \eta \) is total water depth. Subscript \( t \) denotes partial derivative with respect to time, while subscript \( x \) and \( y \) denote spatial derivatives in the \( x \) and \( y \) direction, respectively. Variables \( a_1, a_2, b_1, b_2 \) are defined as

\[
a_1 = \frac{1}{2} \beta^2 - \frac{1}{2}, \quad a_2 = \beta + \frac{1}{2}, \quad b_1 = \frac{1}{2} \beta^2,
\]

\[
b_2 = \beta = \frac{\mathbf{z}}{h} = -0.531
\]

for WNB model equations and \( a_1 = a_2 = b_1 = b_2 = 0 \) for NLSW model equations.

Following [12], Adam-Bashforth scheme is used for predictor step and written as

\[
\Omega_{i,j}^{n+1} = \Omega_{i,j}^n + \frac{h}{2} \left( 23 \Phi_{i,j}^{n+1} - 16 \Phi_{i,j}^n + 5 \Phi_{i,j}^{n-2} \right)
+ 2(\Phi_{i,j})_{n+1} - 3(\Phi_{i,j})_{n-1} + (\Phi_{i,j})_{n-2}
\]

Adam-Moulton scheme is used for corrector step and written as

\[
\Omega_{i,j}^{n+1} = \Omega_{i,j}^n + \frac{h}{2} \left( 9 \Phi_{i,j}^{n+1} + 19 \Phi_{i,j}^n - 5 \Phi_{i,j}^{n-2} + \Phi_{i,j}^{n-3} \right)
+ \Phi_{i,j}^{n+1} - \Phi_{i,j}^n
\]

where \( \Omega = (\eta, U, V) \), \( \Phi = (E, F, G) \) and \( \Phi_1 = (E_1, F_1, G_1) \).

The values of \( u \) and \( v \) at time level \( (n+1) \) could be obtained by solving (10) and (11) using double sweep algorithm to solve tri-diagonal matrix system.

Fig. 1. Staggered grid system for spatial discretization.
A staggered grid system (C grid) in space is used to discretize spatial derivatives as shown in FIG.1. The horizontal velocity vectors \((u, v)\) and sea level \((h)\) are organized into triplets as visualized by triangle in FIG.1. The water depth is defined at the same point of sea level at the cell center, while vectors such as velocity components \(u\) and \(v\) are defined at the interfaces of the cell. At the cell interface, scalars are obtained by linear interpolation. For example, the total water depth at point can be obtained by

\[
\left( h + \eta \right)_{i,j} = \frac{1}{2} \left( h_{i,j} + \eta_{i,j} \right) + \frac{1}{2} \left( h_{i+1,j} + \eta_{i+1,j} \right)
\]  

(15)

Spatial discretizations are required for various orders of spatial derivatives on the right-hand side of (3) and (4) which include first-order, second-order and second-order cross derivatives. The first-order derivative of \(f = (h + \eta)u\) in the \(x\) direction is discretized by four-point finite difference method to eliminates fifth-order physical dispersion in the governing equations. The four-point finite difference scheme is written as

\[
\left( \frac{\partial f}{\partial x} \right)_{i,j} = \frac{f_{i+2,j} - 27f_{i+1,j} + 27f_{i-1,j} - f_{i-2,j}}{24\Delta x}
\]  

(16)

The fourth-order accurate finite difference scheme for first-order space derivative of \(\eta\) at \(u\)-point is

\[
\left( \frac{\partial \eta}{\partial x} \right)_{i,j} = \frac{\eta_{i-1,j} - 27\eta_{i,j} + 27\eta_{i+1,j} - \eta_{i+2,j}}{24\Delta x}
\]  

(17)

First-order derivative of \(u^2\) in the \(x\) direction is finite-differented by the following scheme

\[
\left( \frac{\partial u^2}{\partial x} \right)_{i,j} = \frac{(u^2)_{i-2,j} - 8(u^2)_{i-1,j} + 8(u^2)_{i+1,j} - (u^2)_{i+2,j}}{12\Delta x}
\]  

(18)

Three-point central scheme is used for second order derivatives in the \(x\) direction as

\[
\left( \frac{\partial^2 w}{\partial x^2} \right)_{i,j} = \frac{1}{\Delta x^2} \left[ w_{i-1,j} - 2w_{i,j} + w_{i+1,j} \right]
\]  

(19)

where \(w = u\) or \((hu)\). Similar expressions can be obtained in the \(y\) direction for both first-order and second-order derivatives. The cross-derivative terms in the \(x\) direction of \(u\), \(v\) and \((hu)\) are approximated by the following finite difference scheme

\[
\left( \frac{\partial^2 w}{\partial x \partial y} \right)_{i,j} = \frac{1}{\Delta x \Delta y} \left[ w_{i+1,j} + w_{i,j+1} - w_{i-1,j+1} - w_{i,j-1} \right]
\]  

(20)

Again, \(w = u\) or \((hu)\) and similar expressions can be obtained in the \(y\) direction for \(v\), \((hv)\), and \((hv)\).

### III. Model Simulation and Discussions

#### A. Simulation Settings

Simulations of The Indian Ocean Tsunami 26 December 2004 are conducted using the numerical models. In order to minimize the grid size and achieves resolution but accommodate gauges and satellite data, numerical domain is selected around Bay of Bengal from longitude 70°E–100°E and latitude 7°S–23°N. Bathymetry data is taken from ETOPO1 bathymetry database resulting 1800 by 1800 of grid points with about 1.852 km x 1.852 km grid interval in Cartesian coordinate system. The mean water level specified in the model simulation did not include the effects of tides. Coriolis effect also was not included into the models computation.

According to the grid resolution, time interval was chosen to be 2 seconds due to numerical stability of the models. Radiation boundaries are applied at the south, west and east part of numerical domain by adding sponge layers at the corresponding boundaries. Artificial slot technique for treatment ‘wet-dry’ condition for runup as described in [2] and [5] is used here. The continuity equation (1) must be modified to implement the artificial slot. Detail of implementation of artificial slot technique for runup treatment is referred to [2] and [5]. Constant bottom friction coefficients of 0.001 and 0.0 are applied when the water depth less and greater than 1 km, respectively.

#### B. Generation of Tsunami

Tsunami source model for IOT event has been studied in [3] event. The tsunami source is developed based on rupture (seafloor deformation) parameters which estimated by seismic inversion model and other seismological and geological data. According to rupture trench, the rupture zone is broken into five segments following the trench curvature [3]. Parameter for each segment was characterized and defined by seismic inversion model. The geometry of rupture then estimated by using static dislocation formulae of Okada in [10]. The area of five segments rupture are presented in FIG.2 (left panel).
superposition of the five rupture segments and shown in FIG. 2.
(right panel).
To obtain a good agreement of sea surface along Jason 1’s satellite transect, the five rupture segments are generated in different starting time ($t_0$) and rising time ($T_{rise}$) of vertical seafloor movement based on average shear wave speed about 0.8 km/s from the south to the north. TABLE I presents starting time and rising time of vertical seafloor movement of each rupture segment used in this simulation. FIG. 3 shows comparison of free surface elevations between numerical results and Jason 1’s satellite altimetry data. Model simulation using NLSW model and WNB model show a good agreement with measured data of Jason 1.

C. Dispersion Effects
To investigate the dispersion effect of tsunami propagation, numerical results of NLSW model and WNB model are compared. As the first check to visualize the dispersion effect, snapshot window of sea surface pattern at time 1 hour 40 minutes of tsunami propagation is depicted as shown in FIG. 5. General features of wave evolution results by NLSW model and WNB model are agree very well. However, some differences in reproducing dispersion effect become more noticeable as time advances and longer distance of propagation. The figure shows that the main of tsunami is propagated to the south-west direction, i.e. Maldives islands. Wave pattern at the west part simulated by WNB model is slightly different compare to the NLSW model result. The tsunami front face is shifted and split into more than one wave

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**TABLE I**

Starting Time ($t_0$) and Rising Time ($T_{rise}$) of Seafloor Deformation for Each Segment to Generate Tsunami.

<table>
<thead>
<tr>
<th>Segment 1</th>
<th>Segment 2</th>
<th>Segment 3</th>
<th>Segment 4</th>
<th>Segment 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_0$ (s)</td>
<td>0</td>
<td>185</td>
<td>315</td>
<td>360</td>
</tr>
<tr>
<td>$T_{rise}$ (s)</td>
<td>60</td>
<td>70</td>
<td>90</td>
<td>120</td>
</tr>
</tbody>
</table>

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Fig. 2. Locations of five rupture segment as tsunami source and final form of source elevation as combination of five Okada source determined in [3].

Fig. 3. Comparison of surface elevation measured by Jason 1 satellite altimetry and results of model simulation using NLSW model and WNB model.

Parameters to determine Okada’s formulae for each rupture segment can be obtained in [3]. The five segments of ruptures then used to find the best tsunami source after it is calibrated to the recorded data of Jason 1 satellite altimetry as published in [7]. The final form of seafloor deformation geometry is
yielded a series of wave propagation in which the first one has higher amplitude and longer period than the last one.

The dispersive effect is proportional to the water depth, so the dispersive effect at the west part of the source is stronger compared to the east part. The dispersion effect at the west is also enhanced by the longer distance of propagation. At the east direction, the computation of wave pattern by NLSW model and WNB model is not significantly different. Beside shallowness of water depth at this area, the dispersion effect did not have enough time to develop because of short distance propagation.

Spatial profiles of sea surface at time 3 hours after tsunami propagations along transect line C1 as shown in FIG.4 is presented in FIG.5 to visualize more detail of the dispersion effect. Generally, agreement between the dispersive and non-dispersive model is very good, but after long distance propagation to the south-west direction, the advantage of dispersive model is remarkable. Initially, free surface profiles produced by WNB and NLSW models are not significantly different. But after long time and long distance propagation through relatively deep water, the front face profile is gradually different. The tsunami front face is oscillated and changed by the dispersion effect. According to FIG.5, at time $t = 3$ hours, the WNB model yields at least three waves of tsunami front face at the south-west direction along line C1 with the wavelength of first, second, and third waves are about 141 km, 83 km, and 64 km, respectively, measured from trough to trough. The average water depth is $h \approx 5$ km, therefore the corresponding values of $kh$ for the three waves are 0.2206, 0.3747, and 0.486, respectively. According to the value of $kh$, the second and third waves are categorized as intermediate water wave. Hence, the NLSW model is less accurate for this case. Although the second and third waves are categorized as intermediate water wave, the approximation of dispersive term in WNB model still give accurate estimation of wave speed because the values of $kh < 1$. The leading wave height at this time is over predicted more than 20% by the NLSW model.

At the east part, the agreement between NLSW and WNB model results are quite good. When tsunami wave entered runup phase, nonlinear interaction with complex bathymetry strongly influenced the dispersion effect. It was pointed out in [4] that the dispersion consideration in the numerical models is necessary for accurate prediction for the cases of tsunami entered continental shelf, bays or harbors in which tsunami produced oscillations through the resonance. However, there are no established coastal observations which clearly represent dispersion mechanism.

Following [3], the measured data of tsunami elevations around simulation domain are compared to the model results. Three gauges locations are discussed here, two tide gauges at the Maldives: Hannimadhoo (73.17°E, 6.77°N) and Male (73.54°E, 4.23°N), and one by a Belgian yacht “Merchator” at Nai Harn Bay (SW of Phuket). The measured data is digitized from [3]. Tsunami arrival times are predicted earlier by all of model simulations at all locations compared to observations data. However, generally, the simulated and measured time history of tsunami elevations agree very well in all tide gauges as shown in FIG. 6. It is noted that because of coarseness of bathymetry data used here, the locations of tide gauges are not perfectly match between models and observations.

Fig. 5. Spatial profiles along line C1 at $t = 3$ hours simulated using NLSW model (thick lines) and WNB model (bold lines).

Fig. 6. Comparisons of temporal sea level between measured data as published in [3] and model simulations at: (a) Hannimadhoo (b) Male, and (c) Merchator yacht.
At the Maldives as shown in FIG.6 (a and b), it can be seen a good agreement between observed and model results. General pattern of temporal variation of sea levels are match with observations data for at least three waves. After long distance propagation, the dispersion effect is noticed at those gauges. However, the bathymetry effect is reduced the dispersion effect created by WNB model, hence, similar results are obtained by NLSW model and WNB model but the NLSW model over predicted of maximum height compare to WNB model and measured data.

In FIG.6c, the NLSW and WNB models give the same results. Compare to the observations data of yacht Merchator, the profile is not match. Local coastal topography effect is not resolved very well by the models, so the time shift is different between model results and measurement data. Maximum height is lower predicted by the models. Therefore, finer grid resolution at the east area is necessary to obtain better estimation.

IV. CONCLUSIONS

Numerical simulation of the December 26, 2004 Indian Ocean tsunami has been performed using dispersive and non-dispersive wave models (WNB and NLSW models, respectively). Comparisons of simulation results using the two models notified the dispersion effect especially through oceanic tsunami propagation. Simulation results using the two different model equations showed that the NLSW model is quite reliable for practical purpose because this model gives consistent results compare to WNB model and observations data. However, including the dispersive term will improve accuracy of the prediction. From this study, it can be inferred that it is necessary to use dispersive wave model for modeling of long distance tsunami propagation over relatively deep water, while the non-dispersive wave model could be used for getting quick result, hence, we can select an appropriate model equations for a specific problem.

This study was conducted by using relatively coars grid resolution. For future study, it is necessary to use finer grid resolution to study the effect of dispersion for the cases of tsunami entered continental shelf, bays or harbors in which tsunami produced oscillations through the resonance.

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