Free Vibration Characteristics for Different Configurations of Sandwich Beams

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Abstract— A generalizes model presenting the sandwich beams was developed to calculate the flexural rigidity and sandwich beams dynamic characteristics. Different cases such as sandwich beams multi-layer cores, sandwich beams multi-cells, sandwich beams with holes in its cores having different shapes and different orientations were investigated. The finite element code ANSYS 11 was used for free vibration analysis of the sandwich beams; the natural frequencies and mode shapes and the static deflection of the sandwich beams were calculated. The obtained results from the finite element code ANSYS 11 such as static deflections, static rigidity and natural frequencies were compared with that obtained from the generalized equations according the cases of investigations which appear to be in good agreement with each others, therefore the generalized model can be used for the best design of the sandwich beams.

Index Term— Sandwich beams, Static and Dynamic characteristics, Free vibration, Finite elements, ANSYS

I. INTRODUCTION

To face the challenge of more and more complicated loading conditions, designs of lighter, safer and efficient structures with multi-functuniality have been motivating researchers since a long time. Light weight multi-layered structures and sandwiches also called laminate panels belong to one such kind of advanced structures developed up to now [1]. Lightweight material and structure designs with elegant mechanical properties, e.g., energy absorption, thermal isolation, anti-impact constitute always a challenging work for aerospace, and automotive industries. Geometrically, these structures can be considered as a result of periodic repetitions of a basic cell in one, two or three dimensions[2]. A sandwich material is a layered assembly made from two thin, strong, and stiff face sheets bonded to a lightweight compliant core material. This creates a stiff, strong and also a very lightweight structural element. These structures can carry both in-plane and out-of-plane loads and exhibit good stability under compression, keeping excellent strength to weight and stiffness to weight characteristics. The many advantages of sandwich constructions, the development of new materials and the need for high performance and low-weight structures insure that sandwich construction will continue to be in demand [3-5].

M. Cheheh et al[6] stated that sandwich beams and plates which are categorized as composite structures with high value of strength to weight ratios are considered as new specific advanced structures. Also the free vibration behavior of the sandwich beam with FGM core material was analysed using a meshless method, where the element free Galerkin method as known and robust method is utilized.

As stated by [7] that, the purpose of the core is to maintain the distance between the laminates and to sustain shear deformations. By varying the core, the thickness and the material of the face sheet of the sandwich structures, it is possible to obtain various properties and desired performance.

There are many wide varieties of core materials geometry currently in use. Among them, honeycomb, foam, balsa and corrugated cores are the most widely used. Usually honeycomb cores are made of aluminum or of composite materials: Nomex, glass thermoplastic or glass-phenolic. The other most commonly used core materials are expanded foams, which are often the most to achieve reasonably high thermal tolerance, though thermoplastic foams and aluminum foam are also used. For the bonding of laminate and core materials, normally two types of adhesive bonding are commonly employed in sandwich construction, i.e., co-curing and secondary bonding. Characterization of sandwich materials has been carried out in detail in scientific literature. The determination of the sandwich material behavior under crushing loads and the measurements of the ductile fracture limits is normally done with the help of compression tests[8,9]. Among the various honeycomb sandwich structures, adhesively bonded aluminum honeycomb sandwich panels are currently popular because of their easy assembly of face sheets and cores. One application of such structures is in the cabin sections, such as doors, walls, and floors. Since these sandwich structures are subjected to the dynamic service loading, the progressive fatigue failure are frequently inspected. The homogenized elastic properties of honeycomb sandwich structures with skin effect were subsequently presented by Xu and Qiao [10-12]. As shown in Fig. 1, a variety of sandwich structures with different cell configurations and arrangements are illustrated [1]. In ref [13] Noor et al gave a comprehensive review of different computational models on sandwich panels and shells. Applications were involved in problems of heat transfer, thermal and stresses, free vibration and damping, transient dynamic responses, design optimization, etc. investigated the static deflection of laminated composite beams.

Recently, Hsueh et al.[14] studied the biaxial strength of thin multilayered disks. An analytical model of general closed-form solutions is developed for the elastic stress distributions subjected to biaxial flexure tests. Burgueno et al.[15] studied the designs of hierarchical cellular sandwich beams and plates with high structural efficiency.
experimental tests of cellular plates with different densities of porosity and different hole arrangements over the cross-section, the measured results of effective elastic constant related to the bending stiffness showed a good agreement with the shape factors and material indices. Alternatively, Wang and McDowell [16] and Hayes et al. [17] studied extruded metal honeycombs, i.e., the so-called linear cellular alloys (LCA). The first work was focused on the maximization design of elastic torsion and bending rigidities of the circular sandwich bar structure in terms of the triangular subcell geometry of the sandwich core while the second work was to understand the heat transfer and mechanical behaviors of the circular sandwich beam. The effect of the sandwich beam elements on static and dynamic characteristics of the sandwich beams with single core where different configurations of sandwich beams (different core and face materials and dimensions) were presented and investigated theoretically using developed method and experimentally using the impulse excitation test where good agreement between theoretical and experimental results were obtained.

The vibration characteristics of sandwich materials have drawn much attention recently. The dynamic parameters of a structure, i.e., natural frequency, damping and mode shapes, are determined with the help of vibration testing which provides the basis for rapid and inexpensive dynamic characterization of composite structures[18]. The importance of material damping in the design process has increased in recent years as the control of noise and vibration in high precision, high performance structures. In polymeric composites, the fiber contributes to the stiffness and the damping is enhanced owing to the internal friction within the constituents and interfacial slip at the fiber/matrix interfaces. [19,20] Meng-Kao et al.[21] studied the dynamic properties of sandwich beams through experimental tests and using finite element methods. It was found out that, the faces dominate the stiffness of the sandwich beams, the natural frequencies were affected directly by the face materials and decreases with the increasing fiber orientation of the graphite/epoxy face laminates. Increasing the face of the cores increases both frequencies and loss factors of the sandwich beams.

The aim of the present work is to develop a generalized model presenting the sandwich beams to calculate the flexural rigidity, dynamic characteristics and the characteristics of their free vibrations for different cases such as sandwich beams multi layer cores, sandwich beams multi cells, sandwich beams with holes in its cores having different shapes and different orientations in wrt the sandwich beam axis.

II. THEORETICAL ANALYSIS

Generalized flexural rigidity model for the multi layer cores sandwich beams MLC with (n_c) cores and constant cross section.

$$D = \sum_{i=2}^{n_c+1} E f_i I f_i + \sum_{i=1}^{n_c} E c_i I c_i$$  

.........(1)

Where : $E_f$ and $E_c$ are the Young’s modulus of faces and cores materials respectively.

The cross section moment of inertia for faces $I_f$ and cores $I_{c_i}$ are given by:

$$I_f = (n_c+1) \frac{b h f^3}{12} + \frac{b h f}{4} (h_t + h_c)^2 \sum_{i=1}^{n_c} ((n_c - i)^2), i = 0, 2, 4, ......., n_c$$  

.........(2)

And

$$I_{c_i} = n_c \frac{b h f^3}{12} + \frac{b h_c}{2} (h_t + h_c)^2 \sum_{i=1}^{n_c-1} ((n_c - i - 1)^2), i = 0, 2, 4, ......., n_c$$  

.........(3)

Where: b denotes the beam width, $h_t$ and $h_c$ are the faces and cores thicknesses respectively. The total beam height(H) is therefore given by:

$$H = n_c (h_c + h_t)$$  

.........(4)

Generally, flexural rigidity (D) relationship can be expressed for (n_c) – number of cores (i.e number of cells) as follows:

$$D = \frac{b E f h f^3}{12} [n_c (1 + \frac{E_c}{E_f} \frac{h_c^3}{h f^3}) + 1] + \frac{b E f h f}{2} (h_t + h_c)^2 [\sum_{i=1}^{n_c} ((n_c - i)^2) + \frac{E_c}{E_f} \frac{h c}{h f} \sum_{i=1}^{n_c-1} ((n_c - i - 1)^2)], i = 0, 2, 4, ......., n_c$$  

.........(5)
The investigation of the generalized flexural rigidity function:

1- For sandwich beam with single core:
Let the number of cores \( n_c = 1 \) in single core equation (5) yields to:

\[
D = \frac{bE_fh_f^3}{12} - \left[ (2 + \frac{E_c}{E_f} \frac{h_f^3}{h_f^3}) + \frac{bE_fh_f^3}{2} \right] (h_f + h_c)^2
\]

\( \ldots (6) \)

\[
D = \frac{bE_fh_f^3}{12n_c} \left[ (n_c(1 + \frac{E_c}{E_f} \frac{h_f^3}{h_f^3}) + 1) + \frac{bE_fh_f^3}{2n_c} (h_f + h_c)^2 \right] \sum_{i=1}^{n_c} \left( (n_c - i)^2 + \frac{E_c}{E_f} \frac{h_c}{h_f} \sum_{i=1}^{n_c} ((n_c - i) - i)^2 \right), i = 0, 2, 4, \ldots, n_c
\]

\( \ldots (7) \)

3- The sandwich beams composed of multi cells having the same dimensions.

In this case the sandwich beam can be diagrammatically presented as shown in fig.1 and the flexural rigidity is given as follows:

\[
D = \frac{bE_fh_f^3}{12} \left[ (n_c(1 + \frac{E_c}{E_f} \frac{h_f^3}{h_f^3}) + 1) + \frac{bE_fh_f^3}{2n_c} (h_f + h_c)^2 \right] \sum_{i=1}^{n_c} \left( (n_c - i)^2 + \frac{E_c}{E_f} \frac{h_c}{h_f} \sum_{i=1}^{n_c} ((n_c - i) - i)^2 \right), i = 0, 2, 4, \ldots, n_c
\]

\( \ldots (8) \)

4- The sandwich beams with \( (n_c) \) cores and periodically variable cross-section.

Here, another sort of sandwich beams that have periodically variable cross-sections in the longitudinal or cross direction of the beam axis will be investigated. This can be occur by making hole(s) in the direction or in the cross direction of the beam axis. The aim of this is to control the value of the beam flexure rigidity, mass, natural frequency and modes of vibrations (static and dynamic performance of the beam).

The beam has an upper and bottom face sheets. The core of the unit cell of the beam has a rectangular, square or circular holes. In order to evaluate the effective flexural rigidity, the Boolean operation is used to subtract the contribution of

\[
I_h = n_c \left( \frac{b_h h_f^3}{12} + \frac{b_h h_c}{2} (h_f + h_c)^2 \sum_{i=1}^{n_c} \left( (n_c - i) - i \right)^2, i = 0, 2, 4, \ldots, n_c \right)
\]

For square hole let \( b_h = h_h \) in equation (9)

II- Circular holes in the cores:

\[
I_h = n_c \frac{\pi}{64} b_h h_f^3 + (\frac{\pi}{4}) \frac{b_h h_c}{2} (h_f + h_c)^2 \sum_{i=1}^{n_c} \left( (n_c - i) - i \right)^2, i = 0, 2, 4, \ldots, n_c
\]

Generally equation (9 and 10 ) can be given in a general form as follows:

\[
I_h = n_c \left( a_1 b_h h_f^3 + a_2 \frac{b_h h_c}{2} (h_f + h_c)^2 \sum_{i=1}^{n_c} \left( (n_c - i) - i \right)^2, i = 0, 2, 4, \ldots, n_c \right)
\]

Where :

in case of rectangular or square hole \( [a_1 = 0.083, \ a_2 = 1] \)
and in case of circular hole $a_1 = 0.0156, a_2 = 0.785$.

In this case, the flexure rigidity can be expressed as follows:

$$D = \sum_{i=2}^{n+1} E_{fi} I_{fi} + \sum_{i=1}^{n} E_{ci} I_{cie}$$  

...(12)

Where $I_{cie}$ is the effective moment of inertia of the core(s) which can be calculated as follows:

$$I_{cie} = I_{ci} - I_{hi}$$  

...(13)

And the corresponding flexural rigidity can be calculated as follows:

$$D = \frac{bE_h h_f^3}{12} [n_c (1 + \frac{E_f h_f^3}{E_h h_f'}) + 1] + \frac{bE_h h_f}{2} (h_f' + h_f) \sum_{i=1}^{n} ((n_c - 1) - i)^2 + \frac{E_f h_f}{2} (h_f' + h_f) \sum_{i=1}^{n} ((n_c - 1) - i)^2,$$

$$a_2 \frac{b_h h}{2} (h_f' + h_f) \sum_{i=1}^{n} ((n_c - 1) - i)^2, i = 0,2,4,6,...,n_c$$  

...(14)

$$I_h = n_c \left[ \frac{1}{64} b_h h_c^3 + \frac{b_h h}{2} (h_f' + h_f) \sum_{i=1}^{n} ((n_c - 1) - i)^2 \right]$$

Where the maximum number of holes can be expressed as follows:

$$n_h = \frac{(L - 2d_h)}{2d_h} = \left( \frac{L}{2d_h} - 1 \right)$$  

...(18)

$$D = \frac{bE_h h_f^3}{12} [n_c (1 + \frac{E_f h_f^3}{E_h h_f'}) + 1] + \frac{bE_h h_f}{2} (h_f' + h_f) \sum_{i=1}^{n} ((n_c - 1) - i)^2 + \frac{E_f h_f}{2} (h_f' + h_f) \sum_{i=1}^{n} ((n_c - 1) - i)^2.$$

$$a_2 \frac{b_h h}{2} (h_f' + h_f) \sum_{i=1}^{n} ((n_c - 1) - i)^2, i = 0,2,4,6,...,n_c$$

For $n_c = 1$ and $n_h = 0$, equation (19) can be expressed as follows:

$$D = \frac{bE_h h_f^3}{12} [(2 + \frac{E_f h_f^3}{E_h h_f'}) + h_f^3] + \frac{bE_h h_f}{2} (h_f' + h_f)^2$$  

...(20)

Which was obtained by [5].

**The determination of the dynamic characteristics of multi layer cores MLC sandwich beams:**

The equivalent dynamic characteristics of MLC sandwich beams can be calculated according to its end conditions. In case of cantilever beams supported from one end and free from the other the dynamic characteristics can be expressed as follows:

**A-The equivalent mass ($m_e$):**

$$m_e = (n_c + 1) [b h_f \rho_f] + n_c b h_c \rho_c - V_h \rho_c$$  

.........(21)

Where: $\rho_f$ and $\rho_c$ are the densities of the face core materials, $V_h$ is the volume of the holes in the cores which can be calculated according to its shape, orientation w.r.t. the beam axes and calculated as follows:

- For rectangular or square holes:
\[ V_h = n_c n_f b_h h_c L \]
\[ V_h = 0.785 n_c n_f d_h^2 L \]

Where: \( n_h = 1 \) in case of the holes are in the direction of the beam axes.

\section*{B- The equivalent stiffness \( K_e \):}

\[ K_e = 3 \times 10^3 D / L^3 \]

Where: \( K_e \) is the equivalent stiffness, N/m and the flexural rigidity

\section*{C- The equivalent natural frequency \( f_n \):}

\[ f_n = \frac{1}{2\pi} \sqrt{K_e / (0.25m_e)} \]

The frequencies of different modes can be calculated according to the following equations:

\[ f_i = \frac{\beta^2_i}{2\pi} \sqrt{K_e / (0.25m_e)} \]

Where, \( \beta^2_i \) is a constant dependent on the boundary conditions, and \( i = 1, 2 \ldots n \) are the frequency corresponds to each mode.

\section*{D- The equivalent beam damping coefficient \( C_e \):}

Damping is an important modal parameter for the design of structures for which vibration control and cyclic loading are critical. In sandwich beam structures, the equivalent damping coefficient \( C_e \) is affected by the damping coefficient of the face and core materials. The equivalent sandwich beam damping coefficient is equal to the summation of the total damping coefficient of the faces \( C_f \) and the total damping coefficient of the core \( C_c \) beams and can be expressed as follows:

\[ C_e = \sum C_f + \sum C_c \]

The coefficient of viscous damping of the faces \( C_f \) can be expressed in terms of the critical damping coefficient of the face beam materials as follows:

\[ C_f = \frac{\eta_f k_f}{\pi f_n^2} \]

On the other hand, the coefficient of viscous damping of the core \( C_c \) can be expressed as follows:

\[ C_c = \frac{\eta_c k_c}{\pi f_n^2} \]

Therefore the equivalent coefficient of viscous damping of the MLC sandwich beam \( C_e \) can be expressed as follows:

\[ C_e = \frac{3}{\pi f_n^2} \left[ \eta_f E_f \sum I_f + \eta_c E_c \sum I_c \right] \]

Where \( I_f \) and \( I_c \) can be calculated according to MLC end conditions and the holes orientation in the cores.

\section*{III. RESULTS AND DISCUSSIONS}

The finite element code ANSYS 11 was used for free vibration analysis of the sandwich beams; the natural frequencies and mode shapes and the static deflection of the sandwich beams were calculated. Element Solid45, having eight nodes and six degrees of freedom per node, was used. The obtained results such as static deflections, static rigidity and natural frequencies were compared with that obtained from the generalized equations according the cases of investigations. In the analysis, the material properties of the core and face materials of sandwich beams are \( (E_f = 210 \text{ GPa}, E_c = 4 \text{ GPa}, \rho_f = 7800 \text{ Kg/m}^3, \rho_c = 1200 \text{ Kg/m}^3) \). The sandwich beam width = 20 mm and length L = 300 mm. A perfect bonding at the interface between the face and the core materials was assumed. The sandwich beam is considered to be cantilever type, i.e. fixed at one end.

\section*{1-Comparison between sandwich beam with single core or multi cores keeping the total beam height (H) constant}

Fig. 4 shows the static and modes shapes at response of a sandwich beam of a single core and sandwich beams with multi cores having the same beam height \( H = 16 \text{ mm} \). For this case the thicknesses of the faces and cores are varied according to the number of cores \( n_c \). It is observed that the beams rigidity and hence the beam deflections as well as the 1st, 2nd, 3rd and 4th mode shapes are the same and their frequencies are decreased as presented in Fig. 5. These behaviors can be explained by the influences of the faces
and cores thicknesses on the $I_x$ and $I_y$ of the beams and hence on the beams flexural rigidity(D). The values of constant $\beta_i^2$ which depends on the boundary conditions( fixed- free fixation), are $\beta_1^2 = 1$, $\beta_2^2 = 5.5\beta_3^2 = 13.3\beta_4^2 = 22.4$. A good agreement between the values of static deflections and natural frequencies which obtained from ANSYS 11 with that obtained from theoretical analysis, The obtained results indicate that both of the natural frequency and static rigidity can be adjusted to a certain values by the best selection of the numbers of sandwich beam cores.

2- Sandwich beams composed of multi cells having the same dimensions.

Fig.6, shows the static and modes shapes of a sandwich beam of a single core and sandwich beams with multi identical cells having the same beam height (H). It is observed that as the cell numbers increase, the beam rigidity are increased and hence the beam deflections are decreased as well as the 1st, 2nd, 3rd and 4th mode shapes are the same and their frequencies are increased as presented in Fig.7. These behaviors can be explained by the influences of the faces and cores thicknesses on the $I_x$ and $I_y$ of the beams and hence on the beams flexural rigidity(D).

A good agreement between the values of static deflections and natural frequencies which obtained from ANSYS 11 with that obtained from the theoretical analysis. The obtained results indicate that both of the natural frequency and static rigidity can be adjusted to a certain values by the best selection of the numbers and dimensions of sandwich beam cells.

3- Sandwich beams having holes in its core.

Fig. 8. shows the modes shapes of a sandwich beam of a single core without holes in its core and another one having a square hole in the direction of the sandwich beams axis and a third one having 15 holes in its core in the cross direction of the beam axis. All the sandwich beams having the same beam height (H). It is observed that the beam rigidity, as well as the 1st, 2nd, 3rd and 4th mode shapes and their frequencies are almost the same A good agreement was observed between the values obtained from ANSYS 11 with that obtained from the theoretical analysis.

IV. CONCLUSIONS

From the theoretical analysis and obtained results it can be concluded that:

1- A generalized model is developed. The model is able to investigate the effect of the number of cores, multi cells, and the holes shape and their orientation w.r.t. the beam axis on the static rigidities, natural frequencies and hence on the dynamic properties of the sandwich beams.

2- The finite element code ANSYS 11 was used for free vibration analysis, the natural frequencies, mode shapes and the static deflection of the sandwich beams are calculated.

3- The investigation revealed that the static and dynamic responses of the sandwich beams can be adjusted the increasing of the number of cores or the number of cells.

4- The values of constant $\beta_i^2$ which depends on the boundary conditions( fixed- free fixation), are $\beta_1^2 = 1$, $\beta_2^2 = 5.5\beta_3^2 = 13.3\beta_4^2 = 22.4$. These values are used for calculating of the modes frequency.

5- The obtained results from the finite element code ANSYS 11 such as static deflections, static rigidity and natural frequencies were compared with that obtained from the generalized model according the cases of investigations where good agreement between each other is obtained.

REFERENCES


Sandwich beam with six cores

Fig. 4. Mode shapes for sandwich beams having the same height H=16 mm with different number of cores.

A) Single core

B) Two cores

C) Six cores

Fig. 5. Relationship between number of cores and: A) Flexural rigidity, B) First mode natural frequency, C) Mode frequency for different modes.
Fig. 6. Mode shapes for sandwich beams having with different number of cells.

A) One cell
B) Two cells
C) Four cells

Fig. 7. Relationship between sandwich beams with different number of cells: A) Flexural rigidity, B) First mode natural frequency.
Fig. 8. Mode shapes for sandwich beams with and without holes in the direction of beam axes.